

*This chapter will take approximately 1.25 hours to complete.*

# Basic Gas Properties and Mathematical Manipulations

## OBJECTIVES

### Terminal Learning Objective

At the end of this chapter, the student will be able to identify basic gas properties and mathematical manipulations.

### Enabling Learning Objectives

- 2.1 Differentiate between temperature scales and absolute temperature.
- 2.2 Identify the types of pressure and the instruments used.
- 2.3 Identify the ideal gas laws.
- 2.4 Describe gas density.
- 2.5 Define the term *Reynolds Number*.
- 2.6 Explain equations used to identify basic gas properties.
- 2.7 Identify units of measurements that measure air pollution parameters.
- 2.8 Describe use of references and definitions.

## 2.1 Temperature

### The Fahrenheit and Celsius Scales

The range of units on the Fahrenheit scale between the freezing and boiling point of water at one atmosphere (atm) pressure is 180 (212°F-32°F= 180°F); on the Celsius scale, the range is 100 (100°C-0°C = 100°C). Therefore, each Celsius degree is equal to 9/5 or 1.8 Fahrenheit degrees. To convert from one system to the other, the following equations can be used:

$$\text{(Eq. 2-1)} \quad \text{°F} = 1.8\text{°C} + 32$$

$$\text{(Eq. 2-2)} \quad \text{°C} = \frac{(\text{°F} - 32)}{1.8}$$

Where:            °F = degrees Fahrenheit  
                     °C = degrees Celsius

### Absolute Temperature

Experiments in which a gas volume is determined as a function of temperature (at a constant pressure) yield results similar to the data presented in Figure 2.1(a). The solid portion of each line represents the gaseous state. If each line is extrapolated (dashed portion of line) to a volume of zero, they all intersect at a common temperature (-273.15°C or -459.67°F). This is the temperature at which a gas, if it did not condense, would theoretically have a volume of zero. This temperature (-273.15°C or -459.67 °F) is called *absolute zero*. Another temperature

scale, developed by and named after English physicist Lord Kelvin in 1848, begins at absolute zero and has temperature intervals equal to Celsius units. This absolute temperature scale is in units of *Kelvin* (K). It is not proper to use the term or symbol for “degrees” in association with the *Kelvin scale*. A similar scale was developed, in 1859, by William Rankine, a Scottish engineer and physicist, to parallel the Fahrenheit scale and is called the *Rankine scale* (°R). The following formulas can be used to convert temperatures to their respective absolute scales.

(Eq. 2-3)  $K = ^\circ C + 273.16$

(Eq. 2-4)  $^{\circ}R = ^\circ F + 459.67$

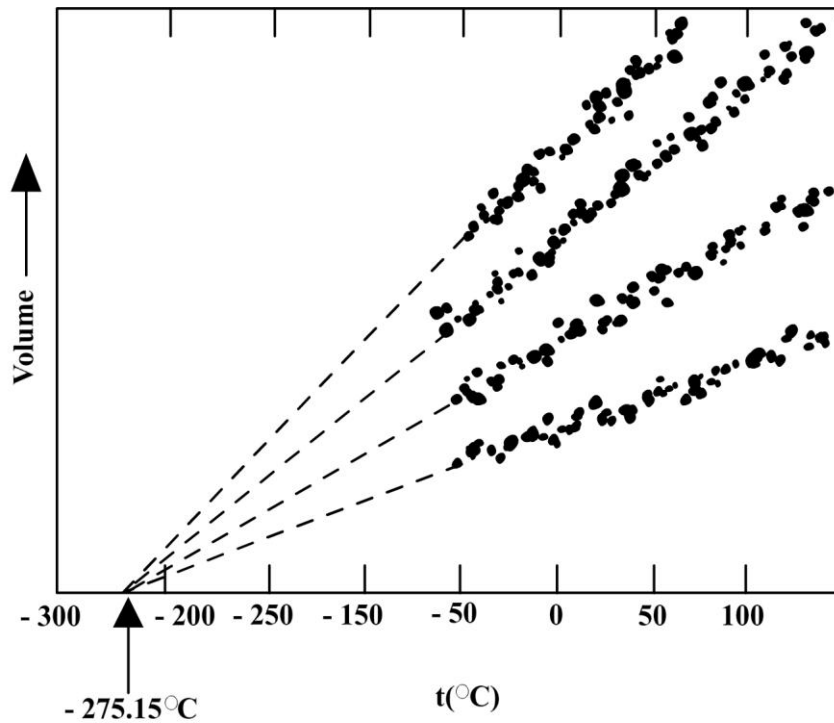


Figure 2-1(a). Temperature relationships.

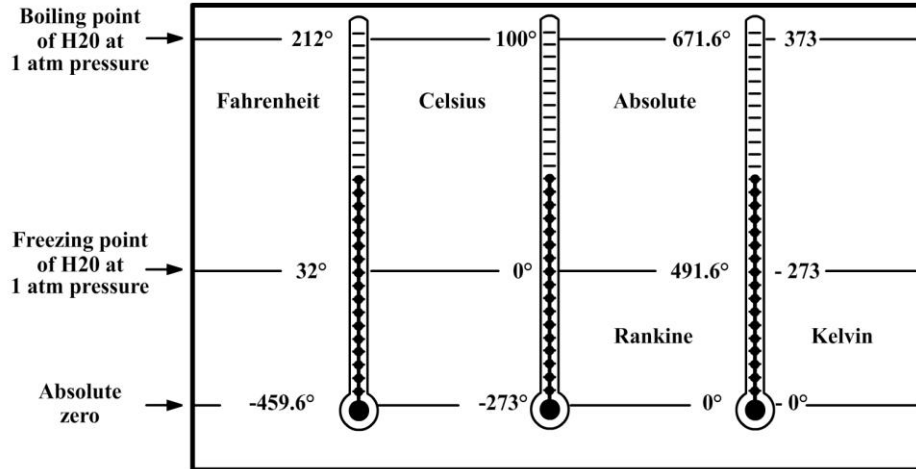


Figure 2-1(b). Relationships of the absolute temperature systems.

Relationships of the absolute temperature systems are shown graphically in Figure 2-1(b). The symbol “T” will be used throughout this manual to denote absolute temperatures, and the “t” will be used to indicate Fahrenheit or Celsius degrees. The absolute temperatures are always in volume calculations involving temperature and pressure.

## 2.2 Pressure

### Definition of Pressure

A body may be subject to three kinds of stress: shear, compression, and tension. Fluids are unable to withstand tensile stress; hence, they are subject to shear and compression only. Unit compressive stress in a fluid is termed *pressure* and is expressed as *force per unit area*.

Pressure	
Metric	English
gm <sub>f</sub> /cm <sup>2</sup>	lb <sub>f</sub> /in <sup>2</sup> (psi)

Pressure is equal in all directions at a point within a volume of fluid and acts perpendicular to a surface.

### Barometric Pressure

Barometric pressure and atmospheric pressure are synonymous. These pressures are measured with a barometer and are usually expressed as inches or millimeters of mercury. Standard barometric pressure is the average atmospheric pressure at sea level, 45° north latitude, at 35°F, and is equivalent to a pressure of 14.696 pounds-force per square inch exerted at the base of a column of mercury 29.921

inches high (in the English System). In the metric system, standard barometric pressure is equivalent to a pressure of 1033.23 grams-force per square centimeter exerted at the base of a column of mercury 760 mm high. Weather and altitude are responsible for barometric pressure variations.

**Torricelli Barometer**

The Torricelli, or mercurial, barometer was first used by one of Galileo’s students, Evangelista Torricelli, in 1643. A mercurial barometer is made by sealing a tube, about 32 inches long, at one end. The tube is filled with mercury. It is then inverted and placed into a container that is partially filled with mercury. The mercury in the tube will fall until the weight of the mercury in the tube is equal to the force of the air pressure on the mercury in the container. As shown in Figure 2-2, the manometer and the mercurial barometer work on the same principle – atmospheric pressure being measured with reference to a vacuum.

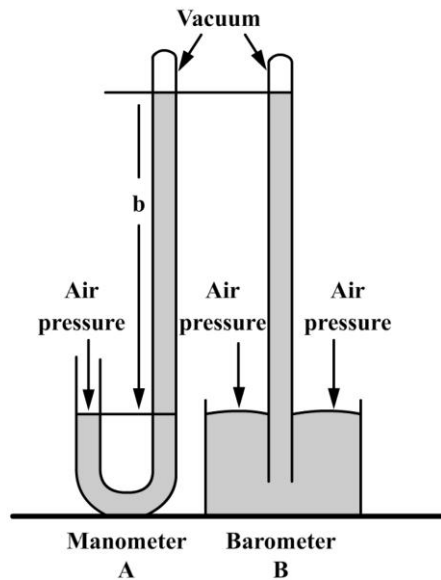


Figure 2-2. The manometer and mercurial barometer.

**Fortin Barometer**

Since the mercurial barometer is the most accurate measurement (calibration uncertainty of 0.001 to 0.03% of reading) of atmospheric pressure, it is still in wide use today. The most common modified version of the mercurial barometer is the Fortin type shown in Figure 2-3.

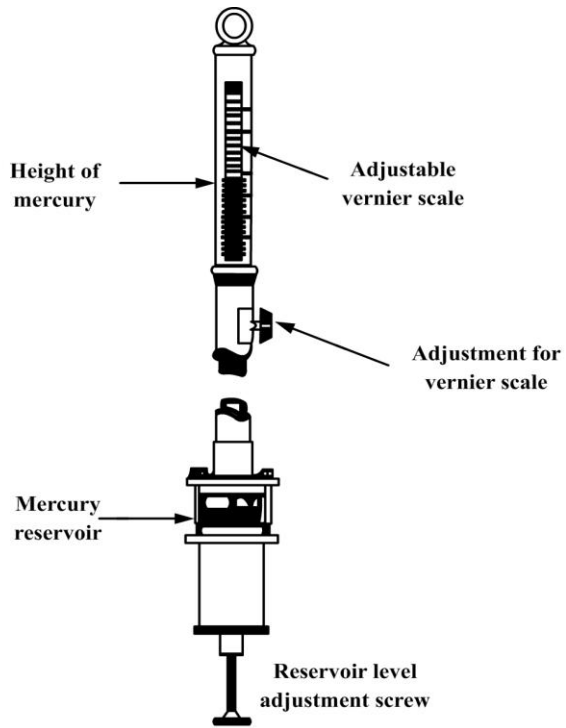


Figure 2-3. Fortin barometer.

The height of the mercury column in a Fortin barometer is measured from the tip of the ivory index point (see the enlargement in Figure 2-4) to the top of the mercury column. The mercury level in the glass cylinder (ambient-vented cistern) is adjusted until the ivory index point just pricks the surface of the mercury. This is done by turning the datum-adjusting screw. Then the vernier scale is adjusted until the bottom of it is even with the top of the mercury meniscus. After the vernier scale is adjusted, the height of the mercury column is read.

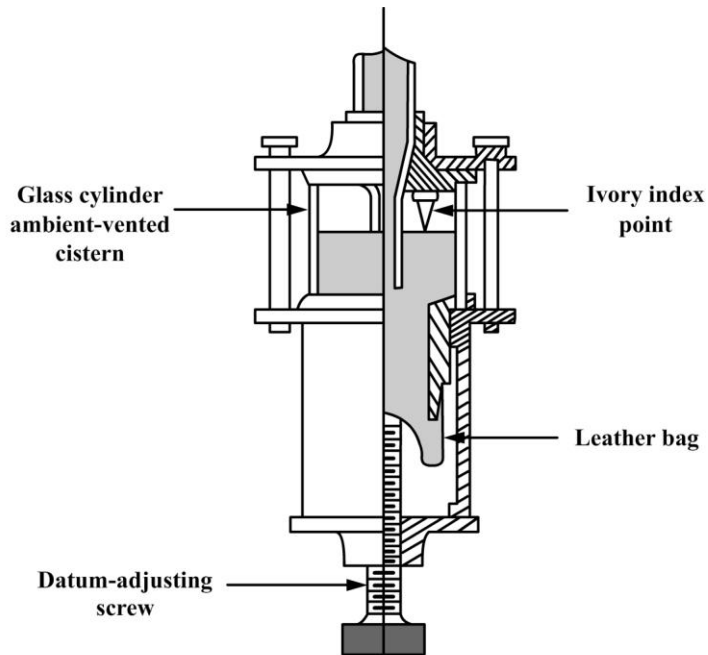


Figure 2-4. Blow-up of Fortin barometer.

A typical vernier scale is shown in Figure 2-5. The barometric pressure indicated in the figure is determined in the following way: the bottom of the vernier scale indicates not only the integer component of the barometric pressure, but also the tenths components – in this case, 29.9. The hundredths component is indicated by the match between the outer scale and the vernier – in this case, 0.04. The readings are totaled to determine the barometric pressure:  $29.9 + 0.04 = 29.94$  in. Hg. The equivalent metric reading is 76.05 cm.

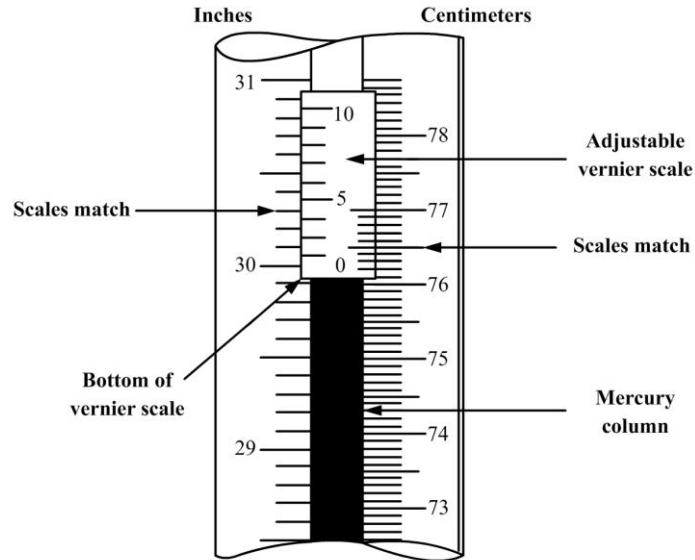


Figure 2-5. Blow-up of vernier scale.

**Aneroid Barometer**

The aneroid barometer is usually not as accurate as a Torricelli barometer. However, aneroid barometers are more widely used because they are smaller, more portable, less expensive, and easier to adapt to recording instrumentation than are Torricelli barometers.

The aneroid barometer usually consists of a metal chamber, bellows, or slyphon (accordion-like) cell that is partially evacuated. A spring is used to keep the metal chamber from collapsing (see Figure 2-6). The width of the chamber is determined by the balance between the spring and the force exerted by the atmosphere. The width of the chamber is indicated by a pointer and scale that can be calibrated to read directly in units of pressure (i.e., millimeters or inches of mercury, etc.). The pointer movement can be amplified by using levers. Read-out systems can vary from visual scales to recording devices. The combination of an aneroid barometer and an automatic recording device is called a barograph.

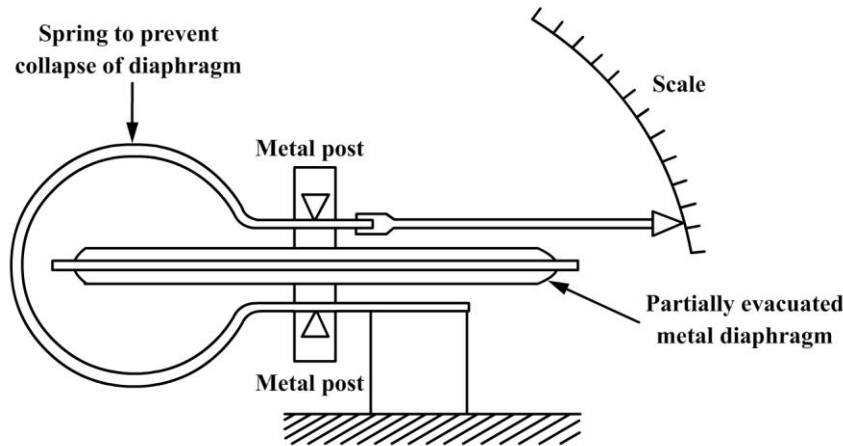


Figure 2-6. Aneroid barometer.

### Pressure Transducers

A transducer is a device that is activated by power from one system and supplies power in some other form to a second system. Conventional pressure transducers use an elastic element that converts the energy from a pressure differential into a displacement of a mechanical device. An example of a mechanical pressure transducer is shown in Figure 2-7. Other pressure transducers convert the mechanical displacement into an electrical signal. An example of an electrical transducer is shown in Figure 2-8. Electrical pressure transducers have become very popular because the signal is easy to measure, control, amplify, transmit, and record.

Other variations on the pressure transducer include the use of the principles of conductivity and capacitance to measure pressure. A *strain gauge* uses the change in conductivity which results from the deflection of a material as it senses pressure changes. The deflection of the sensing material alters its resistance which is related to the change in pressure. Another technology uses the change of capacitance due to change of the distance between charged plates to calculate the pressure.

Pressure sensors can vary drastically in technology, design, performance, application, suitability, and cost. A conservative estimate is that there are 50 different technologies and hundreds of companies making pressure sensors.



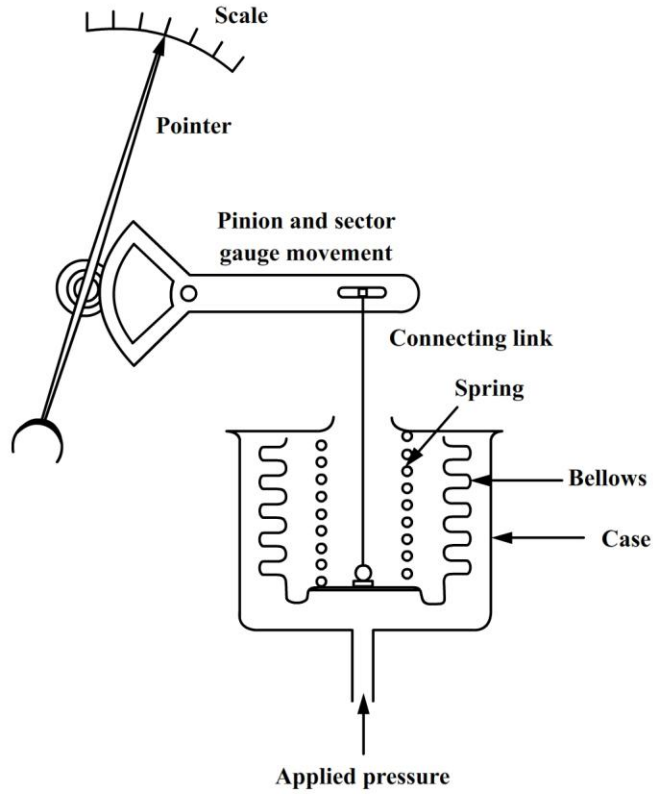


Figure 2-7. Mechanical pressure transducer.

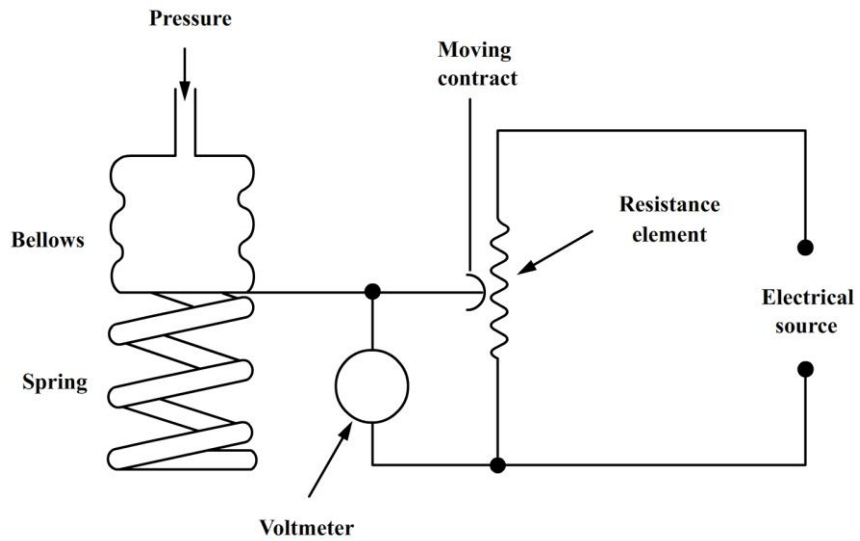


Figure 2-8. Electrical pressure transducer.

**Gauge Pressure**

Gauges indicate the pressure of a system relative to ambient barometric pressure. If the pressure of the system is greater than the pressure prevailing in the atmosphere (atmospheric pressure), the gauge pressure is expressed as a positive value; if smaller, the gauge pressure is expressed as a negative. The term “vacuum” designates a negative gauge pressure. The abbreviation “g” is used to specify a gauge pressure. For example, *psig* means pounds-force per square inch gauge pressure.

The pressure you measure when determining if your car tire is properly inflated is a gauge pressure. In fact, the term “inflated” refers to a system which is pressurized, and as such we know it represents a positive pressure relative to atmospheric pressure. The opposite is true for an evacuated SUMMA flask used to collect an air sample. In this case, the air is drawn out of the flask prior to sampling, creating a negative gauge pressure relative to atmospheric pressure, referred to as a *vacuum*. At the time of sampling, the valve of the flask is opened, and the pressure differential between the atmosphere and the flask (negative relative to the atmosphere) creates the driving force necessary to draw in the sample.

**Absolute Pressure**

Because gauge pressure (which may be either positive or negative) is the pressure relative to the prevailing atmospheric pressure, the gauge pressure, added algebraically to the prevailing atmospheric pressure (which is always positive), provides a value that is called “absolute pressure.” The mathematical expression is:

(Eq. 2-5) 
$$P = P_b + p_g$$

Where:

$P$	=	<i>absolute pressure</i>
$P_b$	=	<i>atmospheric pressure</i>
$p_g$	=	<i>gauge pressure</i>

*Note:*  $P$ ,  $P_b$ , and  $p_g$  must be in the same units of pressure before they can be added (i.e. all must be in inches of mercury, mm of mercury, etc.).

The abbreviation “a” is sometimes used to indicate that the pressure is absolute. For example, *psia* means pounds per square inch absolute pressure. Conventionally, a gauge pressure is denoted with a lower-case “p” and a pressure on an absolute basis with an upper-case “P.”

Equation 2-3 allows conversion of one pressure system to the other. The relationship of the two pressure systems is shown graphically in Figure 2-9 using two typical gauge readings, 1 and 2. Gauge reading 1 is above the prevailing atmospheric pressure and, therefore, is expressed as a positive value. Gauge reading 2 is below the prevailing atmospheric pressure and is expressed as a negative value. Gauge reading 3 has both sides open to the atmosphere; hence, the gauge pressure is zero.

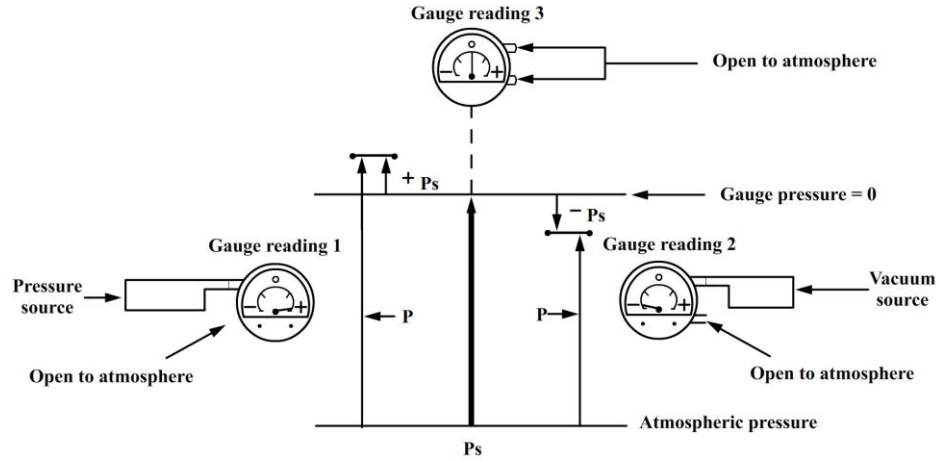


Figure 2-9. Absolute-atmospheric-gauge pressure relationship.

### Example Problems

**Problem 1:**

The primary pressure gauge of a regulator attached to a compressed nitrogen cylinder indicates a reading of 2000 psig. An aneroid barometer mounted on the wall indicates that the atmospheric pressure is 14.2 psi. What is the absolute pressure inside the tank?

Solution:  $P = P_b + p_g = 14.2 + 2000$   
 $P = 2014.2 \text{ psia}$

**Problem 2:**

A water manometer is used to measure the pressure inside an evacuated flask. The water manometer indicates that the evacuated flask has a vacuum of 26 inches of water. A nearby Fortin barometer indicates that atmospheric pressure is 752.6 mm Hg. What is the absolute pressure inside the flask?

Solution:

Before  $p_g$  and  $P_b$  can be added to give  $P$ , both must be in the same unit of pressure. It is most common for  $p_s$  to be converted to the same units as  $P_b$  since  $P_b$  is in much larger units.

Since 1 inch of Hg = 13 inches H<sub>2</sub>O (Hg is 13 times denser than water):

$$(26 \text{ inches H}_2\text{O}) \left( \frac{1 \text{ inch Hg}}{13 \text{ inches H}_2\text{O}} \right) \left( \frac{25.4 \text{ mm}}{1 \text{ inch}} \right) = 50.8 \text{ mm Hg}$$

Now  $p_g$  and  $P_b$  can be added.  $p_g$  is negative because the evacuated flask is at a vacuum, or below atmospheric pressure.

$$P = P_b + \rho_g = 752.6 \text{ mm Hg} + (-50.8 \text{ mm Hg})$$

$$P = 701.8 \text{ mm Hg}$$

**The Concept of Pressure-Head**

Pressure-head is the height of a column of fluid required to produce a given pressure at its base.

The relationship between pressure and pressure-head is:

(Eq. 2-6) 
$$p = \rho_f h \left( \frac{g}{g_c} \right)$$

- Where:
- $p$  = pressure, force/area
  - $\rho_f$  = density of fluid, mass/volume
  - $g$  = local acceleration due to gravity, length/time<sup>2</sup>
  - $g_c$  = dimensional constant
  - $h$  = pressure-head in terms of  $\rho_f$  length

Pressure-head may be expressed in terms of any fluid that is convenient, e.g., Hg or H<sub>2</sub>O.

**Dalton's Law of Partial Pressure**

When gases or vapors (having no chemical interaction) are present as a mixture in a given space, the pressure exerted by a component of the gas-mixture at a given temperature is the same as would be exerted if the gas-mixture filled the whole space alone. *The pressure exerted by one component of a gas-mixture is called its partial pressure. The total pressure of the gas-mixture is the sum of the partial pressures.*

Expressed mathematically:

$$p_{total} = \sum p_i$$

- Where:
- $p_{total}$  = total pressure exerted by the system
  - $p_i$  = pressure of each component of the system
  - $\sum p_i$  =  $p_1 + p_2 + \dots + p_{19}$
  - ( $\sum$  = sum of)

## 2.3 The Ideal Gas Laws

Ideal gases are gases whose molecules do not attract one another and which occupy no part of the whole volume. Although there are no gases which have these properties, real gases, which deviate very slightly from ideal gas behavior under ordinary temperature and pressure conditions, may be considered to be ideal gases.

### Boyle's Law

Boyle's Law states: when the *temperature* ( $T$ ) is held constant, the *volume* ( $V$ ) of a given mass of an ideal gas of a given composition varies inversely as the absolute pressure ( $P$ ), i.e.:

$$V \propto \frac{1}{P} \text{ (at constant } T\text{)}$$

( $\propto$  = proportional to)

One can see that, as the pressure on a gas system increases, the volume of the gas system will decrease and vice versa.

### Charles' Law

Charles' Law states: when the *pressure* ( $P$ ) is held constant, the *volume* ( $V$ ) of a given mass of an ideal gas of a given composition varies directly as the *absolute temperature* ( $T$ ), i.e.:

$$V \propto T \text{ (at constant } P\text{)}$$

In other words, as the temperature of a gas system increases, the volume will also increase and vice versa.

### The Law for Ideal Gases

Both Boyle's and Charles' Laws are satisfied in the following equation:

(Eq. 2-7)  $PV = nRT$

<i>Where:</i>	$P$	= absolute pressure
	$V$	= volume of a gas
	$T$	= absolute temperature
	$R$	= universal gas-constant
	$n$	= number of moles of a gas

A *mole* of a substance is the substance's molecular weight, expressed in mass units, where the substance's molecular weight is the sum of the atomic weights of the atoms which compose the substance.

We know that

$$n = \frac{m}{M}$$

Where:  $m$  = mass of a gas  
 $M$  = molecular weight of a gas

therefore: 
$$PV = nRT = \frac{m}{M} RT$$

The units of R depend upon the units of measurement used in the equation. Some useful values are:

- (1)  $R = 0.082 \text{ (L) (atm) (K)}^{-1} \text{ (g-mole)}^{-1}$
- (2)  $R = 62.4 \text{ (L) (mm Hg) (K)}^{-1} \text{ (g-mole)}^{-1}$

Where the units are:

- $V$  (L)
- $m$  (g)
- $M$  (g/g-mole)
- $T$  (K)
- $P$  [atm for (1) or mm Hg for (2)]

Different values of R can be obtained by utilizing the appropriate conversion factors.

**Molar Volume ( $\bar{V}$ )**

One mole of any gas at 273 K (0° C) and 760 mm Hg will occupy 22.414 liters. This constant is obtained from the ideal gas law. From Equation 2-5:

If:  $P = 760 \text{ mm Hg}$   
 $n = 1 \text{ mole}$   
 $R = 62.4 \text{ (L) (mmHg) (g-mole)}^{-1} \text{ (K)}^{-1}$   
 $T = 273 \text{ K}$   
 $V = V \text{ (molar volume)}$

then: 
$$P \times V = n \times R \times T$$

$$(760 \text{ mm Hg}) (V) = (1 \text{ g-mole}) \left( 62.4 \frac{\text{L} \cdot \text{mm Hg}}{\text{g-mole K}} \right) (273 \text{ K})$$

$$V = \frac{(1)(62.4)(273)}{760} \ell$$

$$V = 22.414 \text{ L} = \bar{V}$$

Therefore, one mole of an ideal gas at 273 K and 760 mm Hg occupies

22.414 liters (L). In other words, the molar volume ( $\bar{V}$ ) of an ideal gas at 273 K and 760 mm Hg is 22.414 L/mole.

At EPA standard conditions (760 mm Hg, 298 K), one mole of any gas will occupy 24.46 L. The volume per mole constant for any gas at a given pressure and temperature is called the *molar volume* and is symbolized by  $\bar{V}$ .

## 2.4 Gas Density

Finally, we can use the ideal gas law to estimate gas density. Density is the ratio of the mass of a material to the volume that material occupies. For accurate values, gas densities should be determined from reference texts. However, an estimate of the gas density can be determined from the ideal gas law. Recognizing that the number of moles is given by mass ( $m$ ) divided by molecular weight (MW), the ideal gas law may be written:

$$\begin{aligned} \text{(Eq. 2-8)} \quad PV &= nRT = \frac{m}{M}RT \\ \frac{m}{V} &= \rho = \frac{PM}{RT} \end{aligned}$$

Where:

$\rho$	= density
$P$	= absolute pressure
$M$	= molecular weight
$T$	= absolute temperature
$R$	= universal gas constant

Another method of determining density is by utilizing the fact that there are 24.46 liters per g-mole at 298 K and 760 mm Hg.

In the relationship  $\rho = m/V$ , if  $V$  is in terms of molar volume,  $\bar{V}$  (L/g-mole of a gas at STP), then  $m$  must be in terms of molecular weight,  $M$ (g/g - mole). So  $\rho = m/\bar{V}$  at a given temperature and pressure.

$$\rho = \frac{M \frac{298}{T} \frac{P}{760}}{24.46}, \text{ corrected to standard temperature and pressure conditions}$$

Where:

$\rho$	= gas density (g/L)
$M$	= molecular weight (g/g-mole)
24.46	= molar volume (L/g-mole)
298	= temperature (K) at standard conditions
$T$	= temperature (K) at actual conditions
760	= pressure (mm Hg) at standard conditions

$P$  = pressure (mm Hg) at actual conditions

**Standard Conditions for Atmospheric Sampling**

To be able to compare gas sampling data collected by various agencies and other organizations, all gas volumes must be corrected to a set of predetermined (“standard”) conditions. For atmospheric or ambient sampling, these conditions are:

25°C or 298 K, and 760 mm Hg.

The equation used to correct volumes sampled to standard conditions is:

(Eq. 2-9)

$$V_2 = (V_1) \left( \frac{P_1}{P_2} \right) \left( \frac{T_2}{T_1} \right)$$

$$V_2 = (V_1) \left( \frac{P_1}{760 \text{ mm Hg}} \right) \left( \frac{298 \text{ K}}{T_1} \right)$$

$$V_2 = (V_1) (0.39) \left( \frac{P_1}{T_1} \right)$$

Where:

- $V_2$  = volume of gas at 2nd conditions or at  $P_2$  and  $T_2$ , L
- $V_1$  = volume of gas at 1st conditions of  $P_1$  and  $T_1$ , L
- $T_1$  = initial temperature of gas, K
- $T_2$  = final temperature of gas--in this case, 298 K
- $P_1$  = initial pressure of gas, mm Hg
- $P_2$  = final pressure of gas--in this case, 760 mm Hg
- $0.39 = \frac{298}{760}$

The term “standard conditions for temperature and pressure” is abbreviated STP.

**Origin and Definition of Viscosity**

Viscosity is the result of two phenomena: (a) intermolecular cohesive forces and (b) momentum transfer between flowing strata caused by molecular agitation perpendicular to the direction of motion. Between adjacent strata of a flowing fluid a shearing stress results that is directly proportional to the velocity gradient (Figure 2-10). *Viscosity is often defined as resistance to flow.*

The relationship of these forces is shown in Equation 2-8.

(Eq. 2-10)

$$g_c T = \mu \frac{dv}{dy}$$

Where:

- $g_c$  = dimensional constant
- $T$  = unit shearing stress between adjacent layers of fluid



$$\frac{dv}{dy} = \text{velocity gradient}$$

$$\mu = \text{proportionality constant (viscosity)}$$

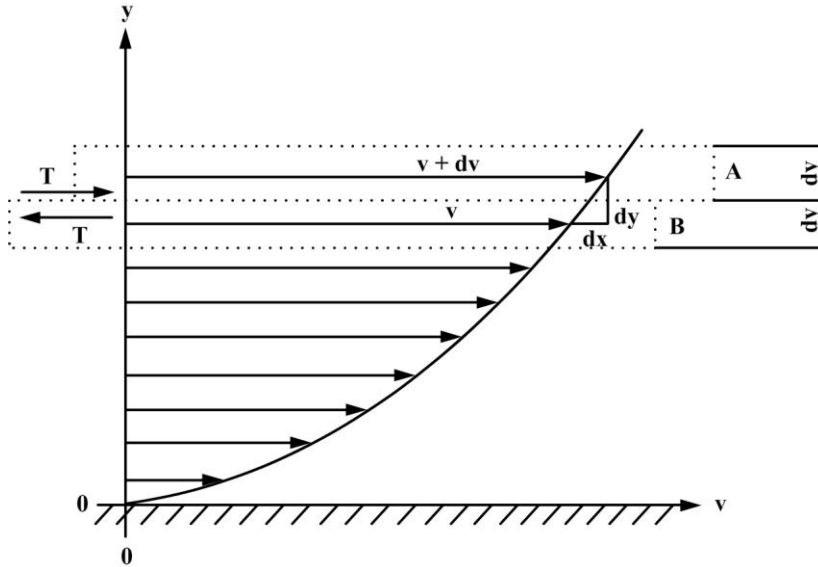


Figure 2-10. Velocity gradient.

The proportionality constant,  $\mu$ , is called the coefficient of viscosity, or merely *viscosity*. It should be noted that the pressure does not appear in Equation 2-10, indicating that the shear ( $T$ ) and the viscosity ( $\mu$ ) are independent of pressure. (Viscosity actually increases very slightly with pressure, but this variation is negligible in most engineering problems.)

### Kinematic Viscosity

Kinematic viscosity is defined according to the following relationship:

(Eq. 2-11) 
$$v = \frac{\mu}{\rho}$$

Where:

- $v$  = kinematic viscosity
- $\mu$  = viscosity of the gas
- $\rho$  = density of the gas (note the absence of dimensions of force)

### Liquid Viscosity versus Gas Viscosity

#### *Liquid Viscosity*

In a liquid, transfer of momentum between strata having different velocities is small, compared to the cohesive forces between the molecules. Hence, shear stress ( $T$ ) is predominantly the result of intermolecular cohesion. Because forces of cohesion decrease with an increase in temperature, the shear stress decreases with an increase in temperature. Equation 2-10 shows that shear stress is directly proportional to the viscosity. Therefore, liquid viscosity decreases when the temperature increases.

#### *Gas Viscosity*

In a gas, the molecules are too far apart for intermolecular cohesion to be effective. Thus, shear stress is predominantly the result of an exchange of momentum between flowing strata caused by molecular activity. Because molecular activity increases with temperature increases, the shear stress increases with a rise in the temperature. Therefore, gas viscosity increases when the temperature increases.

### Determination of Viscosity of Gases

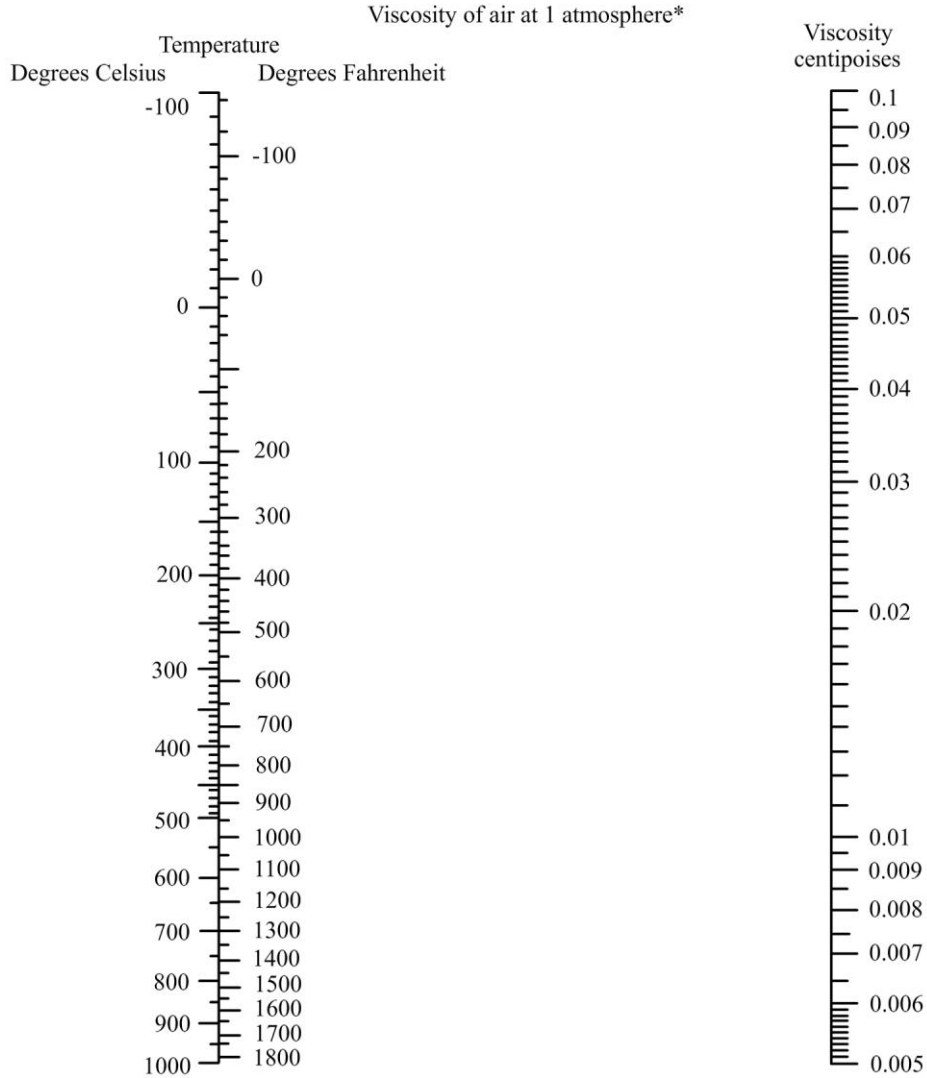
The viscosity of a gas may be found accurately from the following formula:

$$\text{(Eq. 2-12)} \quad \frac{\mu}{\mu^{\circ}} = \left( \frac{T}{273.1} \right)^n$$

Where:

$\mu$	=	viscosity at temperature $T$ (K)
$\mu^{\circ}$	=	viscosity at $0^{\circ}\text{C}$ and prevailing pressure
$T$	=	absolute prevailing temperature (K)
$n$	=	an empirical exponent ( $n = 0.768$ for air)

The viscosity of air and other gases at various temperatures and at a pressure of 1 atmosphere can be determined from the nomograph in Figures 2-11 and 2-12, or from Equation 2-10. The unit of the viscosity coefficient is the poise: 1 poise = gm/cm•sec. A centipoise (cp) is equal to  $10^{-2}$  poise.



(1)centipoise	$(10)^{-2} \frac{\text{gm}}{\text{cm} \cdot \text{sec}}$ $(10)^{-2}$ poise	$209(10)^{-6}$ $\frac{1\text{b} \cdot \text{sec}}{\text{ft}^2}$	$2.09(10)^{-6}$ $\frac{\text{alug}}{\text{ft} \cdot \text{sec}}$	$6.72(10)^{-4}$ $\frac{1\text{b}_m}{\text{ft} \cdot \text{sec}}$
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\*Perry, J.H. Chemical engineer's Handbook. McGraw-Hill Book Co., New York (1950)

Figure 2-11. Viscosity nomograph for air.

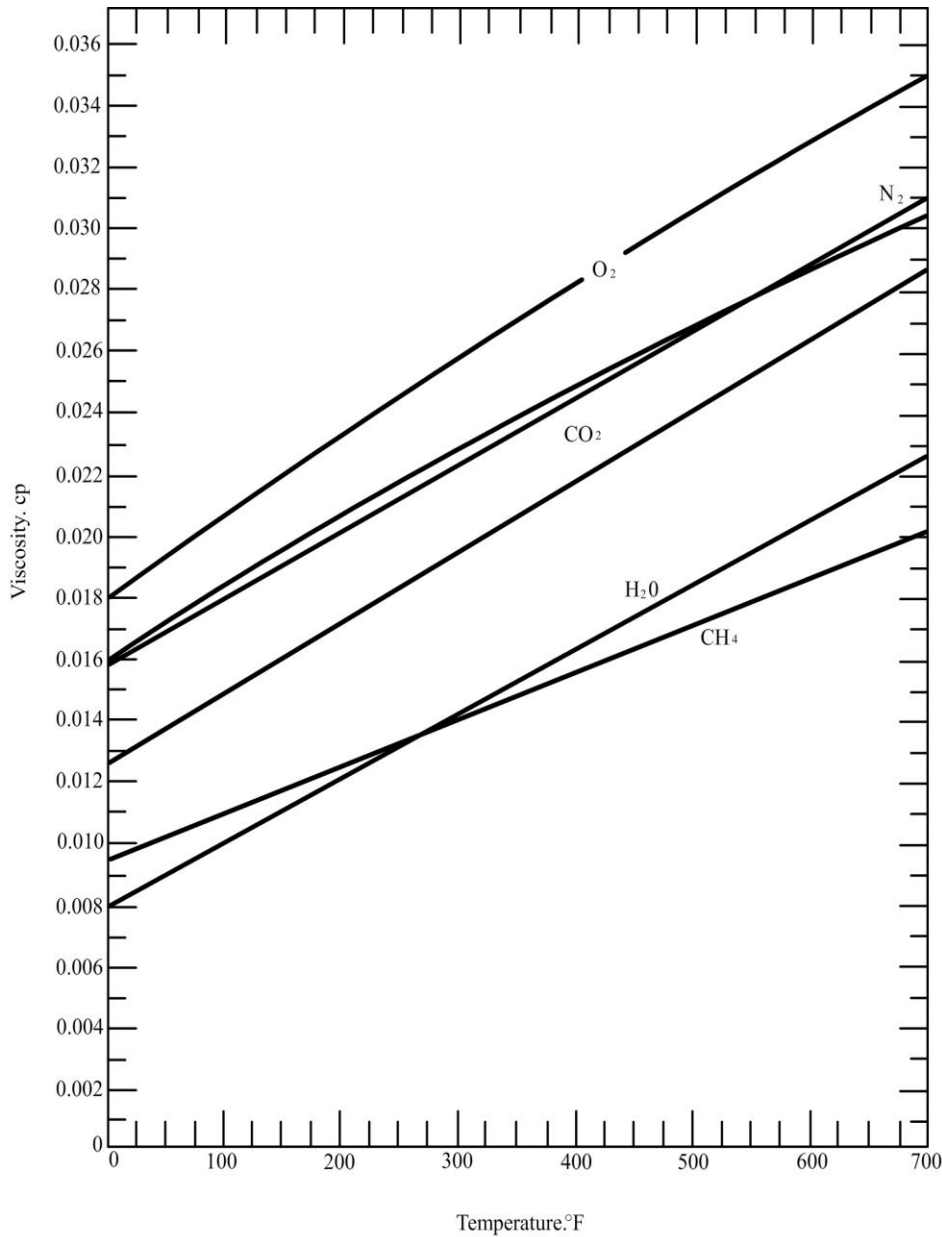


Figure 2-12. Viscosity nomograph of various gases at 1 atmosphere.

## 2.5 Reynolds Number

In 1883, Osborne Reynolds experimented with laminar and turbulent flow. His basic experiment was to inject a dye in a small section of fluid flowing in a tube and find where the flow changed from laminar to turbulent flow. He found that the flow could change abruptly and it could be predicted based on the ratio of inertial forces to viscous forces in a flowing fluid. A typical inertial force per unit

volume of fluid is  $\rho v^2/L$ . A typical viscous force per unit volume of fluid is  $\mu v/L^2$ . The first expression divided by the second provides the dimensionless ratio known as the Reynolds Number:

$$\text{(Eq. 2-13)} \quad Re = \frac{Lv\rho}{\mu}$$

Where:

$Re$	=	Reynolds Number
$L$	=	characteristic system dimension
$v$	=	fluid velocity
$\rho$	=	fluid density
$\mu$	=	fluid viscosity

The linear dimension,  $L$ , is a length characteristic of the flow system. It is equal to four times the mean hydraulic radius, which is the cross-sectional area divided by the wetted perimeter. Thus, for a circular pipe,  $L$  is the pipe diameter ( $D$ ) and the Reynolds Number (sometimes termed the Flow Reynolds Number) takes the form:

$$\text{(Eq. 2-14)} \quad Re = \frac{Dv\rho}{\mu}$$

Reynolds Number in this form is used to distinguish between laminar and turbulent flow. In *laminar flow*, the fluid is constrained to motion in layers, or laminae, by the action of viscosity. These layers of fluid move in parallel paths that remain distinct from one another (i.e., no mixing of molecules between layers). Laminar flow occurs when the Reynolds Number is less than about 2,000. In *turbulent flow*, the fluid is not restricted to parallel paths but moves forward in a random, chaotic manner. Fully turbulent flow occurs when the Reynolds Number is greater than about 3,000. Between Reynolds Numbers of 2,000 and 3,000, the flow may be laminar or turbulent, depending on the flow system conditions. Pipe or duct vibration, for example, can cause turbulent conditions to exist at Reynolds Numbers significantly below 3,000.

Another form of Reynolds Number is the Particle Reynolds Number. Here, the characteristic system dimension is the particle diameter,  $d_p$ , and the velocity,  $v_p$ , and describes the particle velocity relative to the gas stream:

$$\text{(Eq. 2-15)} \quad Re_p = \frac{D_p v_p \rho}{\mu}$$

The Particle Reynolds Number is used to characterize flow conditions when particles move through or with a flowing fluid. Particle Reynolds Numbers less than about one indicate laminar conditions and define what is commonly termed the *Stokes Region*. Values over about 1,000 indicate turbulent conditions and define what is commonly termed the *Newton Region*. Particle Reynolds Numbers between 1 and 1,000 indicate transitional conditions. Most particle motion in air pollution control devices occurs in the Stokes and Transitional Regions.

Reynolds Number determination for flow conditions and particle behavior may be important considerations when designing air sampling equipment and instrumentation, but it is generally not of concern if the equipment is being utilized within its design parameters. For example, an impactor may be designed to capture particles which are  $< 2.5 \mu\text{m}$ , as an aerodynamic diameter, at a flow rate of 16.7 liters per minute. If the flow rate differs appreciably from the design flow rate, the size of the particles being captured will be altered as well. The concepts of Flow and Particle Reynolds Numbers are also important to the design and evaluation of air pollution control equipment and are discussed in more detail in EPA Course 413, Control of Particulate Emissions. The Flow Reynolds Number as it relates to cyclonic flow determinations is a topic of discussion in EPA Course 450, Source Sampling for Particulate Emissions.

## 2.6 Summary of Useful Equations

### Temperature

$$^{\circ}\text{C} = (^{\circ}\text{F} - 32)/1.8$$

$$^{\circ}\text{F} = 1.8^{\circ}\text{C} + 32$$

$$\text{K} = ^{\circ}\text{C} + 273$$

Where:

$^{\circ}\text{F}$	=	<i>degrees Fahrenheit</i>
$^{\circ}\text{C}$	=	<i>degrees Celsius</i>
$\text{K}$	=	<i>degrees Kelvin</i>

### Pressure

$$P = P_b + p_g$$

$$P = \rho_f \left( \frac{g}{g_c} \right) b$$

$$\rho_{f(1)} b_{f(1)} = \rho_{f(2)} b_{f(2)}$$

$$1 \text{ std atm} = 29.92 \text{ in. Hg} = 760 \text{ mm Hg}$$

Where:

$P$	=	<i>absolute pressure</i>
$p$	=	<i>pressure</i>
$\rho$	=	<i>density</i>
$b$	=	<i>pressure-head or height</i>
$g$	=	<i>gravitational acceleration</i>
$g_c$	=	<i>dimensional constant</i>

Subscripts:

$g$	=	<i>gauge</i>
$f$	=	<i>fluid</i>
$b$	=	<i>barometric or atmospheric</i>

**Ideal Gas Law**

$$PV = nRT$$

$$PV = \frac{m}{M} RT$$

$$R = \left( \frac{0.08205(\text{liters})(\text{atm})}{(\text{g-mole})(\text{K})} \right) = \frac{62.4(\text{liters})(\text{mm Hg})}{(\text{g-mole})(\text{K})}$$

1 g-mole = 22.414 liters at 273 K and 760 mm Hg (molar volume)

Where:

$P$	=	absolute pressure
$V$	=	volume
$m$	=	mass
$M$	=	molecular weight
$R$	=	gas constant
$T$	=	absolute temperature
$n$	=	number of g-moles of a gas

**Gas Density**

$$\rho = \frac{PM}{RT}$$

Where:

$\rho$	=	density
$P$	=	absolute pressure
$R$	=	gas constant
$T$	=	absolute temperature
$M$	=	molecular weight.

**Viscosity,  $\mu$**

$$1 \text{ cp} = 6.72 \times 10^{-4} \frac{\text{lb}_m}{\text{ft} \cdot \text{sec}}$$

$$1 \text{ poise} = 1 \frac{\text{g}}{\text{cm} \cdot \text{sec}}$$

$$1 \text{ cp} = 10^{-2} \text{ poise}$$

**Reynolds Number**

$$Re = \frac{L\rho v}{\mu} = \frac{\text{inertial force}}{\text{viscous force}}$$

Where:

$\rho$	=	density of the fluid (mass/ volume)
$r$	=	velocity of the fluid

$g_c$	=	<i>dimensional constant</i>
$L$	=	<i>a linear dimension</i>
$\mu$	=	<i>viscosity of the fluid</i>
$Re$	=	<i>Reynolds Number</i>

## 2.7 Units of Measurements

### Recommended Units

At the present time, air pollution personnel are confronted with a multitude of confusing and conflicting units of expression. A search through the literature has shown a wide variation in the methods of reporting data. Many of the units of expression are carryovers from other fields, such as water pollution studies and industrial hygiene surveys. While these methods of expression are correct, their application to air pollution studies is often misleading. This section of the manual covers the units presently being used and those recommended for the more commonly measured air pollution parameters.

The recommended units were selected so that the reported values would be small whole numbers in the metric system. If possible, the reported units should be the same as those that are actually measured. For example, weight should be reported in grams or milligrams, and volume in cubic meters. The measured value should never be multiplied by large numbers to extrapolate to extremely large areas or volumes. If this is done, the resulting values are misleading. For example, to report particulate fallout on a weight per square mile basis, the area actually sampled, which is about 1 square foot, would have to be extrapolated to a square mile by multiplying the measured results by almost 28,000,000. Reporting the results on the basis of a square mile is misleading, because we are saying that the 1 square foot that we sampled is representative of a square mile surrounding this sampling site. This we know, in most cases, is not true.

When reporting results, the type of sampling instrument should be described, and when volumes of air are sampled, the temperature and pressure at the time of the sampling should be reported.



Table 2-1. Measurement units.

Parameter	Units in use	Unit recommended	Typical range
Particle fallout	Tons per square mile per month Tons per square mile per year Pounds per acre per month Pounds per acre per year Pounds per thousand square feet per month Ounces per square foot per month Grams per square foot per month Grams per square meter per month Kilograms per square kilometer per month Grams per month per 4-inch or 6-inch jar Milligrams per square inch per month Milligrams per square centimeter per time interval	mg/cm <sup>2</sup> /mo (or yr)	0.5 to 135 mg/cm <sup>2</sup> /mo
Outdoor airborne particulate matter	Milligrams per cubic meter Parts per million by weight Grams per cubic foot Grams per cubic meter Micrograms per cubic meter Micrograms per cubic foot Pounds per thousand cubic feet	µg/m <sup>3</sup> (std)	10 to 5000 µg/ m <sup>3</sup> as TSP  30 to 140 µg/ m <sup>3</sup> as PM <sub>10</sub> <sup>1</sup>  5 to 20 µg/ m <sup>3</sup> as PM <sub>2.5</sub> <sup>1</sup>
Gaseous materials	Milligrams per cubic meter Micrograms per cubic meter Micrograms per liter Parts per million by volume Parts per hundred million Parts per billion by volume Parts per trillion by volume Ounces per cubic foot Pounds per cubic foot Grams per cubic foot Pounds per thousand cubic feet	ppm or ppb by volume  or µg/m <sup>3</sup> (std)	Parts per trillion to parts per million  --
Standard conditions for reporting gas volumes	760 millimeters Hg pressure and 20°C 760 millimeters Hg pressure and 0°C 760 millimeters Hg pressure and 65 °F 760 millimeters Hg pressure and 25°C 700 millimeters Hg pressure and 0°C 700 millimeters Hg pressure and 20°C 30 inches of mercury pressure and 65°F	760 mm Hg and 25°C	--
Particle counting	Number per cubic meter of gas Number per liter of gas Number per cubic centimeter of gas Number per cubic foot of gas	Number of particles/m <sup>3</sup> of gas	10 million and above particles/m <sup>3</sup>

<sup>1</sup> Based on National Trends data for PM<sub>10</sub> & PM<sub>2.5</sub>.

**APTI 435: ATMOSPHERIC SAMPLING COURSE**

Particle count in sedimentation devices (horizontal and vertical)	--	Number of particles/cm <sup>2</sup> /mo (or yr)	--
Temperature	Degrees Celsius Degree Fahrenheit	°C	--
Time	12:00 a.m. to 12:00 p.m. 0000 to 2400	0000 to 2400 (military time)	--
Pressure	Atmospheric pressure Atmospheres Millimeters of mercury Inches of mercury Sampling pressure Millimeters of mercury Inches of mercury Millimeters of water Inches of water	mm Hg	--
Sampling rates	Cubic meters per second Cubic meters per minute Cubic feet per second Cubic feet per minute Liters per second Liters per minute Cubic centimeters per second Cubic centimeters per minute	m <sup>3</sup> /min. liters per minute	L/min to 3 m <sup>3</sup> /min
Visibility	Miles and fractions of a mile Kilometers and fractions of a kilometer	km	--

**Conversion Problems**

*Sample Gas Problem*

**Discussion:** The expression parts per million is without dimensions, i.e., no units of weight or volume are specifically designated. Using the format of other units, the expression may be written:

$$\frac{\text{parts}}{\text{million parts}}$$

“Parts” are not defined. If “cubic centimeters” replace “parts,” we obtain:

$$\frac{\text{cubic centimeters}}{\text{million cubic centimeters}}$$

Similarly, we might write pounds per million pounds, tons per million tons,

or liters per million liters. In each expression, identical units of weight or volume appear in both the numerator and denominator and may be cancelled out, leaving a dimensionless term.

An analog of parts per million is the more familiar term “percent.” Percent can be written:

$$\frac{\text{parts}}{\text{hundred parts}}$$

To convert from parts per million by volume, ppm ( $\mu\text{L}/\text{L}$ ), it is necessary to know the molar volume at the given temperature and pressure and the molecular weight of the pollutant. At  $25^\circ\text{C}$  and 760 mm Hg, one mole of any gas occupies 24.46 liters.

**Convert the following:** 2.5 ppm by volume of  $\text{SO}_2$  was reported as the atmospheric concentration.

- What is this concentration in micrograms ( $\mu\text{g}$ ) per cubic meter ( $\text{m}^3$ ) at  $25^\circ\text{C}$  and 760 mm Hg?
- What is the concentration in  $\mu\text{g}/\text{m}^3$  at  $37^\circ\text{C}$  and 752 mm Hg?

**Solution:** Let parts per million equal  $\mu\text{L}/\text{L}$ , then  $2.5 \text{ ppm} = 2.5 \mu\text{L}/\text{L}$ . The molar volume at  $25^\circ$  and 760 mm Hg is 24.46L, and the molecular weight of  $\text{SO}_2$  is 64.1 g/mole.

$$\text{a. } \frac{2.5\mu\text{L}}{\text{L}} \times \frac{1\mu\text{mole}^*}{24.46\mu\text{L}} \times \frac{64.1\mu\text{g}}{\mu\text{mole}} \times \frac{1000\text{L}}{\text{m}^3} = 6.5 \times 10^3 \frac{\mu\text{g}}{\text{m}^3} \text{ at STP}$$

\*Since, at STP, 1 mole of a gas occupies 24.46 liters,  $1 \mu\text{mole} = 24.46 \mu\text{L}$ .

$$\text{b. } (24.46\mu\text{L}) \left( \frac{310 \text{ K}}{298 \text{ K}} \right) \frac{760 \text{ mm Hg}}{752 \text{ mm Hg}} = 25.73\mu\text{L}$$

$$\frac{2.5\mu\text{L}}{\text{L}} \times \frac{1\mu\text{mole}}{25.73\mu\text{L}} \times \frac{64.1\mu\text{g}}{\mu\text{mole}} \times \frac{1000\text{L}}{\text{m}^3} = 6.2 \times 10^3 \frac{\mu\text{g}}{\text{m}^3} \text{ at } 37^\circ\text{C}, 752 \text{ mm Hg}$$

This sample problem also points out the need for reporting temperature and pressure when the results are presented on a weight-to-volume basis.

### Problems

- Convert the following:
  - $68^\circ\text{F} - ^\circ\text{C}$  (answer:  $20^\circ\text{C}$ )
  - $28^\circ\text{C} - \text{K}$  (answer: 301 K)
  - 29.03 in. Hg – mm Hg (answer: 737.3 mm Hg)

2. An ideal gas occupies a volume of 2000 mL at 700 mm Hg and 20°C. What is the volume of the gas at STP? (answer: 1874 mL)
3. If a concentration of carbon monoxide (CO) is noted as 10 ppm, what is this concentration in terms of  $\mu\text{g}/\text{m}^3$  at STP (CO = 28 g/mole)? (answer: 11,440  $\mu\text{g}/\text{m}^3$ , 11.4  $\text{mg}/\text{m}^3$ )
4. Ambient air was sampled at a rate of 2.25 liters per minute for a period of 3.25 hours at 19°C, 748 mm Hg. What volume of air was sampled at STP? (answer: 441 L)
5. Convert 1000  $\mu\text{g}/\text{m}^3$   $\text{SO}_2$  at STP to ppm ( $\text{SO}_2 = 64$  g/mole). (answer: 0.38 ppm)

Tables to use in this task appear in Appendix C of this manual.

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