Theory and Calibration Procedures for the Use of a Rotameter

F.1 Nomenclature

\[ A_f = \text{cross-sectional area of the float} \]
\[ A_m = \text{annular area between the circumference of the float and the inside circumference of the meter tube at that position} \]
\[ C = \text{drag coefficient} \]
\[ C_m = \text{length which is characteristic of the physical system under study (used to calculate Reynolds Number)} \]
\[ d = \text{length which is characteristic of the physical system under study} \]
\[ D_f = \text{diameter of the float} \]
\[ D_t = \text{diameter of the tube at the float position} \]
\[ g = \text{local acceleration due to gravity} \]
\[ g_\text{c} = \text{dimensional constant} \]
\[ m_f = \text{mass of the float} \]
\[ M_m = \text{molecular weight of the metered gas} \]
\[ M_1, M_2, M_3...\text{etc.} = \text{value of molecular weight of the metered gas at conditions 1, 2, 3...etc.} \]
\[ Re = \text{Reynolds Number} \]
\[ Re/C_m = \text{dimensionless factor defined by Equation F-14} \]
\[ P_m = \text{absolute pressure of the metered gas} \]
\[ P_1, P_2, P_3...\text{etc.} = \text{values of absolute pressure at conditions 1, 2, 3...etc.} \]
\[ Q_m = \text{volumetric flow rate through the meter at conditions of pressure (P_m), temperature (T_m), and molecular weight (M_m)} \]
\[ R = \text{universal gas constant} \]
\[ T_m = \text{absolute temperature of the metered gas} \]
\[ T_1, T_2, T_3...\text{etc.} = \text{values of absolute temperature at conditions 1, 2, 3...etc.} \]
\[ v = \text{average gas velocity through the annular area of the meter} \]
\[ V_f = \text{volume of the float} \]
\[ \mu = \text{viscosity of flowing fluid (used to calculate Reynolds Number)} \]
\( \mu_m \) = viscosity of the metered gas
\( \rho \) = density of flowing fluid (used to calculate Reynolds Number)
\( \rho_f \) = density of the float
\( \rho_m \) = density of the metered gas

### F.2 Description of a Rotameter

The rotameter (Figure F-1) is a variable area meter which consists of a vertical, tapered, transparent tube containing a float. The float moves upward as the fluid flow increases. A variable ring or annulus is created between the outer diameter of the float and the inner wall of the tube. As the float moves upward in the tube, the area of the annulus increases. The float will continue to move upward until a pressure drop across the float, which is unique for each rotameter, is reached. This pressure drop across the float is constant regardless of the flow rate. Graduations are etched on the side of the tube so that an instantaneous reading may be observed.

![Figure F-1. Rotameter.](image)
F.3 Development of Flow Equations

General Flow Rate Equations
A free body diagram of the forces acting upon the rotameter float is shown in Figure F-2. The weight of the float is equal to the force of gravity acting on the float. The buoyant force is equal to the weight of the gas that is displaced by the float. The drag force is equal to the frictional forces acting between the float and the moving gas stream.

Mathematically, these forces are as follows:

\[ \text{Drag force} = \frac{CA_f \rho g v^2}{2g_c} \]
\[ \text{Weight of float} = \frac{V_f \rho g}{g_c} \]
\[ \text{Buoyant force} = \frac{V_f \rho_m g}{g_c} \]

Where:
\[ A_f = \text{cross sectional area of the float} \]
\[ C = \text{drag coefficient} \]
\[ g = \text{local acceleration due to gravity} \]
\[ g_c = \text{dimensional constant} \]
When the forces acting in an upward direction exactly equal the force acting in a downward direction, the float will remain stationary in the tube. Equating these forces yields:

\[ \frac{CA_f Q_m v^2}{2g_c} + \frac{V_f Q_m g}{g_c} = \frac{V_f Q_f g}{g_c} \]

Cancelling like terms \((g)\) and rearranging yields:

\[ V_f Q_f g - V_f Q_m g = \frac{CA_f Q_m v^2}{2} \]

Solving for \(v\) and factoring out \(V_f\) and \(g\) from the first two terms yields:

(Eq. F-1)

\[ v = \left[ \frac{2V_f g \left( Q_f - Q_m \right)}{CA_f} \right]^{1/2} \]

The area of the float is equal to \(\pi D_f^2 / 4\), where \(D_f\) is the diameter of the float. Substituting \(\pi D_f^2 / 4\) for \(A_f\) in Equation F-1 yields:

(Eq. F-2)

\[ v = \left[ \frac{8V_f g \left( Q_f - Q_m \right)}{C\pi D_f^2 Q_m} \right]^{1/2} \]

Let \(C_m\) equal \((8/\pi)^{1/2}\), where \(C_m\) is called a meter coefficient and is dependent on the drag coefficient. Substituting \(C_m\) for \((8/\pi)^{1/2}\) in Equation F-2 yields:

(Eq. F-3)

\[ v = C_m \left[ \frac{V_f g \left( Q_f - Q_m \right)}{D_f^2 Q_m} \right]^{1/2} \]

Because the drag coefficient \(C_f\) is dependent on Reynolds Number, \(C_m\) must also be a function of Reynolds Number. Because the density of the gas flowing in the rotameter is very small compared to the density of the float, it can be
ignored in the \((Q_f - Q_w)\) term. Modifying the \((Q_f - Q_w)\) term in Equation F-3 yields:

\[
\rho \cdot \bar{v} = C_\omega \left[ \frac{V_f g Q_f}{D_f^2 Q_w} \right]^{1/2}
\]

\((\text{Eq. F-4})\)

The volumetric flow rate (\(Q_v\)) through the rotameter is equal to the product of the velocity (\(\bar{v}\)) and the annular area of the meter (\(A_w\)). Substituting \(Q_w / A_w\) for \(\bar{v}\) in Equation F-4 yields:

\[
\frac{Q_w}{A_w} = C_\omega \left[ \frac{V_f g Q_f}{D_f^2 Q_w} \right]^{1/2}
\]

Rearranging terms and removing \(D_f^2\) from the radical yields:

\[
Q_w = C_\omega A_w \left[ \frac{V_f g Q_f}{Q_w} \right]^{1/2}
\]

\((\text{Eq. F-5})\)

The density of the float \(\rho_f\) is equal to the mass of the float (\(m_f\)) divided by the volume of the float. Substituting \(m_f / V_f\) for \(\rho_f\) in Equation F-5 and cancelling the \(V_f\)s yields:

\[
Q_w = C_\omega A_w \left[ \frac{g m_f}{Q_w} \right]^{1/2}
\]

\((\text{Eq. F-6})\)

The density of the gas mixture passing through the meter (\(\rho_w\)) is equal to \(P_w M_w / RT_w\), where \(P_w\) is the absolute pressure at the meter, \(M_w\) is the apparent molecular weight of the gas mixture passing through the meter, \(R\) is the universal gas constant, and \(T_w\) is the absolute temperature of the gas mixture. Substituting \(P_w M_w / RT_w\) for \(\rho_w\) in Equation F-6 yields the general flow rate equation for a rotameter:

\[
Q_w = C_\omega A_w \left[ \frac{g m_f \sqrt{T_w}}{P_w M_w} \right]^{1/2}
\]

\((\text{Eq. F-7})\)

**Computation of Reynolds Number**

Reynolds Number is defined as \(\gamma d \rho / \mu\), where \(\gamma\) is the velocity flow, \(d\) is a length which is characteristic of the physical system under study, \(\rho\) is the density of the flowing fluid, and \(\mu\) is the viscosity of the flowing fluid. When
calculating Reynolds Number for a gas flowing through a rotameter, the length characteristic of the physical system \( d \) is the difference between the tube diameter \( D \) and the diameter of the float \( D_f \). Therefore, Reynolds Number may be calculated by using the following equation:

\[
Re = \frac{\nu (D_f - D)}{\mu}
\]

The average velocity of flow through the rotameter is given by \( \frac{Q_m}{A_m} \) where \( Q_m \) is the volumetric flow rate through the meter and \( A_m \) is the annular area between the inside circumference of the tube at the float position.

Substituting \( \frac{Q_m}{A_m} \) for \( \nu \) in Equation F-8 yields:

\[
Re = \frac{Q_m (D_f - D)}{A_m \mu}
\]

The density of the flowing fluid \( \rho \) is equal to \( \frac{P_m M_m}{RT_m} \), where \( P_m \) is the absolute pressure of the metered gas, \( M_m \) is the apparent molecular weight of the metered gas, \( R \) is the universal gas constant, and \( T_m \) is the absolute temperature of the metered gas.

Substituting \( \frac{P_m M_m}{RT_m} \) for \( \rho \) in Equation F-9 yields:

\[
Re = \frac{Q_m (D_f - D) P_m M_m}{A_m \mu RT_m}
\]

Adding the subscript \( m \) to the viscosity term \( \mu \) in Equation F-10 to denote the viscosity of the metered gas yields the following equation, which is used to calculate Reynolds Number for gas flow in a rotameter.

\[
Re = \frac{Q_m (D_f - D) P_m M_m}{A_m \mu_m RT_m}
\]


It can be seen from Equation F-7 that the volumetric flow rate through a rotameter can be calculated when such physical characteristics as the diameter and the mass of the float and the annular area of the meter at each tube reading are known, providing measurements are made of the temperature, pressure, and molecular weight of the metered gas. Before these calculations of the
volumetric flow rate can be made, data must be known about the meter coefficient, \( C_m \). The meter coefficient being a function of Reynolds Number is ultimately a function of the conditions at which the meter is being used. To obtain data on the meter coefficient, the meter must be calibrated. However, because of the ease involved in using calibration curves, common practice is to use calibration curves to determine volumetric flow rates instead of calculating the flow rates from raw data.

**Procedures for the Calibration of a Rotameter**

A common arrangement of equipment for calibrating a rotameter is shown in Figure F-3.

Flow through the calibration train is controlled by the metering valve. At various settings of the rotameter float, measurements are made of the flow rate through the train and of the pressure and temperature of the gas stream at the rotameter. The temperature of the gas stream is usually assumed to be the same as the temperature of the ambient air. If the test meter significantly affects the pressure or temperature of the gas stream, measurements should also be made of the actual pressure and temperature at the test meter. A typical rotameter calibration curve is illustrated in Figure F-4.
To make the calibration curve useful, the temperature and pressure of the volumetric flow rate must be specified.

**A Universal Calibration Curve**

The normal arrangement of the components in a sampling train is shown in Figure F-5. Since the meter is usually installed downstream from the pollutant collector, it can be expected to operate under widely varying conditions of pressure, temperature, and molecular weight. This requires a different calibration curve for each condition of pressure, temperature, and molecular weight. This can be facilitated by drawing a family of calibration curves, which would bracket the anticipated range of pressures, temperatures and molecular weights, as shown in Figure F-6.
Operation of a rotameter under extreme sampling conditions, particularly extreme temperatures, complicates the calibration setup. It is difficult, if not impossible, for most laboratories to be able to calibrate flow metering devices at high temperatures or unusual gas mixtures (especially where toxic gases are involved). For these reasons, it is desirable to develop a calibration curve which is independent of the actual expected sampling conditions. As previously mentioned, the flow through a rotameter is dependent upon the value of $C_m$, the meter coefficient (see Equation F-7), which is a function of the Reynolds Number for the flow in the rotameter. Therefore, to be independent of the sampling conditions, the calibration curve must be in terms of $C_m$ and $Re$.

**Development of a Universal Calibration Curve**

Solving Equation F-7 for $C_m$ gives the following relationship:

\[
C_m = \frac{Q_w D_f}{A_w} \left( \frac{P_w M_w}{g m \mu_w RT_w} \right)^{1/2}
\]

(Eq. F-12)

Dividing Equation F-11 by Equation F-12 yields:

\[
\frac{Re}{C_m} = \frac{Q_w (D_r - D_f) P_w M_w}{A_w \mu_w RT_w}
\]

\[
= \frac{Q_w D_f}{A_w} \left( \frac{P_w M_w}{g m \mu_w RT_w} \right)^{1/2}
\]

Cancelling the like terms $Q_w$ and $A_w$ yields:
\[
\frac{Re}{C_m} = \frac{(D_r - D_f)P_m M_m}{\mu R T_f}\left(\frac{P_m M_m}{\mu R T_f}\right)^{1/2}
\]

Simplifying:

\[
\frac{Re}{C_m} = \left[\frac{(D_r - D_f)P_m M_m}{\mu R T_f}\right] \left[\frac{1}{D_f} \left(\frac{\mu R T_f}{P_m M_m}\right)^{1/2}\right]
\]

Combining the like terms \(P_m\), \(M_m\) and \(T_m\) yields:

(Eq. F-13)

\[
\frac{Re}{C_m} = \left[\frac{(D_r - D_f)P_m M_m}{\mu D_f}\right] \left(\frac{\mu R T_f}{P_m M_m}\right)^{1/2}
\]

Simplifying the \((D_r - D_f)\) and \(D_f\) relationship in Equation F-13 yields a dimensionless factor which has no limitations on either Reynolds Number or the meter coefficient \(C_m\).

(Eq. F-14)

\[
\frac{Re}{C_m} = \frac{1}{\mu} \left[\frac{D_r}{D_f}\right] \left(\frac{\mu R T_f}{P_m M_m}\right)^{1/2}
\]

A plot of the dimensionless factor \(Re/C_m\) defined by Equation F-14 versus the meter coefficient \(C_m\) as calculated from Equation F-12 on regular graph paper will yield a universal calibration curve which is independent of the sampling conditions. Such a plot is illustrated in Figure F-7.
F.5 Use of the Universal Calibration Curve for a Rotameter

To Determine an Existing Flow Rate
To determine an existing flow rate, measurements must be made of the gas temperature and pressure as well as the float position. Data from the manufacturer of the rotameter will yield information on the diameter of the tube at the various float positions and on the diameter and mast of the float. The apparent molecular weight of the gas being metered can be calculated if the composition of the gas stream is known. The viscosity of the gas stream can be determined if the temperature of the gas stream is known (see Perry’s Chemical Engineer’s Handbook). From this data the $Re/C_m$ factor (see Equation F-14) can be calculated. The universal calibration curve is then entered at the calculated value of $Re/C_m$ and the corresponding $C_m$ is noted. $Q_m$ is then calculated from Equation F-7.

To Establish a Required Sampling Rate
To establish a required sampling rate, estimates are made of the metered gas pressure ($P_m$), the metered gas temperature ($T_m$), the apparent molecular weight of the metered gas ($M_m$), and the area of the meter ($A_m$) which will exist at the desired sampling rate. Using these estimated values, the meter coefficient, $C_m$, is calculated (see Equation F-12) for the desired sampling rate.
rate $Q_w$. The universal calibration curve (see Figure F-7) is entered at this value of $C_w$ and the corresponding factor is noted. \[ \left[ \frac{D_r}{D_f} \right] - 1 \] is solved by using the following equation which is a rearrangement of Equation F-14:

(Eq. F-15)
\[
\left[ \frac{D_r}{D_f} \right] - 1 = \mu_w \left( \frac{Re}{C_m} \right) \left( \frac{RT_w}{g m_f P_m M_w} \right)^{\frac{1}{2}}
\]

The float position can be determined from the value \[ \left[ \frac{D_r}{D_f} \right] - 1 \]. For some rotameters the value \[ \left[ \frac{D_r}{D_f} \right] - 1 \] is the tube reading divided by 100. If the area of the meter corresponding to this float position is not equal to the original estimated value for the meter area, the new value of area is used as an estimate and the entire procedure is repeated until the estimated area and the calculated area are equal. Then upon setting the float position at this tube reading, $T_w$, $P_w$, and $M_w$, are noted. If they are different from the original estimates, the procedure is repeated using the observed values of $T_w$, $P_w$, and $M_w$ as estimates. Experience will aid in selecting original estimates that are nearly accurate so that the required sampling rate may be set fairly rapidly.

**To Predict Calibration Curves**

The above techniques are very cumbersome to apply in the field and, as a result, the universal calibration curve should not be used in such a manner.

The real utility of the universal calibration curve is that it can be used to predict calibration curves at any set of conditions. This results in a great reduction in laboratory work in that the rotameter need only be calibrated once and not every time the conditions at which the meter is operated change.

The first step in predicting calibration curves from the universal calibration curve of a rotameter is to ascertain the anticipated meter operating range for the sampling application of concern. Once this operating range is established, an arbitrary selection of a point on the universal calibration curve is made (see point in Figure F-8). The coordinates of point a, point b $(Re/C_m)$, and point c $(C_m)$ are determined. Values of $T_i$, $P_i$, and $M_i$ and the value of $Re/C_m$ are used to calculate a value for \[ \left( \frac{D_r}{D_f} \right) - 1 \] by means of the following equation:

(Eq. F-15)
\[
\left[ \frac{D_r}{D_f} \right] - 1 = \mu_w \left( \frac{Re}{C_m} \right) \left( \frac{RT_w}{g m_f P_m M_w} \right)^{\frac{1}{2}}
\]
Figure F-8. Predicting calibration curves from the universal calibration curve (N\textsubscript{Re} = Re, or Reynolds Number).

The area of the meter (\( A_m \)) is calculated from this value of \( [(D_t/D_f) - 1] \) and is used along with the assumed values of \( T_1, P_1, \) and \( M_1 \) and the value of \( C_w \) from the universal calibration curve to calculate a volumetric flow rate by means of the following equation:

\[
Q_m = \left( \frac{C_w A_m}{D_f} \right) \left( \frac{g m f RT_m}{P_w M_w} \right)^{1/2}
\]

(Eq. F-7)

This procedure is repeated until enough points are available to plot a normal calibration curve. The entire procedure is repeated using new values for temperature, pressure, and molecular weight until a family of calibration curves is plotted. Of course, this family of curves should bracket the anticipated meter operating conditions for the sampling application of concern. The volumetric flow rate (\( Q_m \)) is plotted versus either the area of the meter (\( A_m \)) or the tube reading that corresponds to the meter area.

Field operation is greatly simplified if the tube reading is used. A typical family of calibration curves is shown in Figure F-9.
Notice that these curves are similar to the calibration curves illustrated in Figure F-6. The difference between them is the manner in which they were obtained. The curves of Figure F-6 were obtained by an actual laboratory calibration run for each set of conditions illustrated, whereas the curves of Figure F-9 were obtained by mathematical manipulation of data from only one calibration run. This can, of course, save considerable laboratory time. In addition, it may not be possible to ascertain, in the laboratory, calibration data at extreme conditions, particularly at high temperatures.