Appendix F

Theory and Calibration Procedures for the Use of a Rotameter

F.1 Nomenclature

A_f	=	cross-sectional area of the float
A_m	=	annular area between the circumference of the float and the inside circumference of the meter tube at that position
C	=	drag coefficient
C_m	=	length which is characteristic of the physical system under study (used to calculate Reynolds Number)
d	=	length which is characteristic of the physical system under study
D_f	=	diameter of the float
D_t	=	diameter of the tube at the float position
g	=	local acceleration due to gravity
g_c	=	dimensional constant
m_f	=	mass of the float
M_m	=	molecular weight of the metered gas
M_1, M_2, M_3...etc.	=	value of molecular weight of the metered gas at conditions 1, 2, 3...etc.
Re	=	Reynolds Number
Re/C_m	=	dimensionless factor defined by Equation F-14
P_m	=	absolute pressure of the metered gas
P_1, P_2, P_3...etc.	=	values of absolute pressure at conditions 1, 2, 3...etc.
Q_m	=	volumetric flow rate through the meter at conditions of pressure (P_m), temperature (T_m), and molecular weight (M_m)
R	=	universal gas constant
T_m	=	absolute temperature of the metered gas
T_1, T_2, T_3...etc.	=	values of absolute temperature at conditions 1, 2, 3...etc.
v	=	average gas velocity through the annular area of the meter
V_f	=	volume of the float
μ	=	viscosity of flowing fluid (used to calculate Reynolds Number)

μ_m = viscosity of the metered gas
ϱ = density of flowing fluid (used to calculate Reynolds Number)
ϱ_f = density of the float
ϱ_m = density of the metered gas

F.2 Description of a Rotameter

The rotameter (Figure F-1) is a variable area meter which consists of a vertical, tapered, transparent tube containing a float. The float moves upward as the fluid flow increases. A variable ring or annulus is created between the outer diameter of the float and the inner wall of the tube. As the float moves upward in the tube, the area of the annulus increases. The float will continue to move upward until a pressure drop across the float, which is unique for each rotameter, is reached. This pressure drop across the float is constant regardless of the flow rate. Graduations are etched on the side of the tube so that an instantaneous reading may be observed.

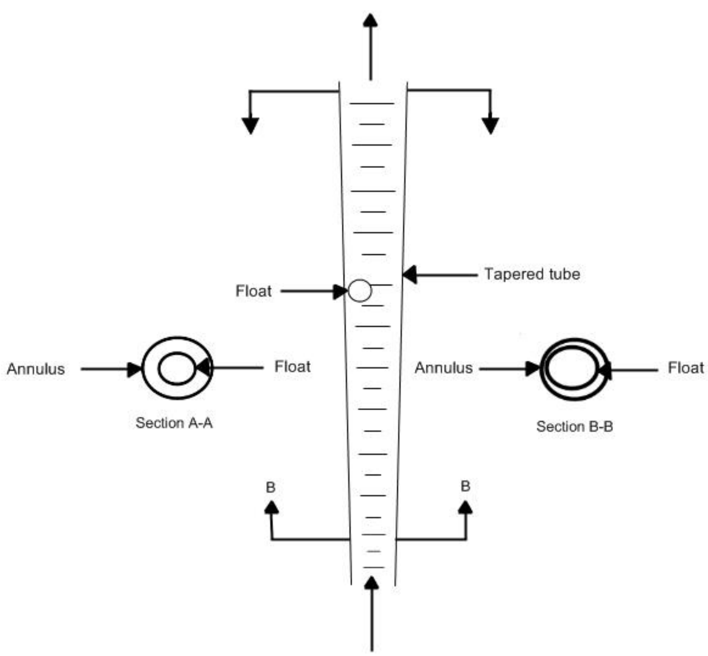


Figure F-1. Rotameter.

F.3 Development of Flow Equations

General Flow Rate Equations

A free body diagram of the forces acting upon the rotameter float is shown in Figure F-2. The weight of the float is equal to the force of gravity acting on the float. The buoyant force is equal to the weight of the gas that is displaced by the float. The drag force is equal to the frictional forces acting between the float and the moving gas stream.

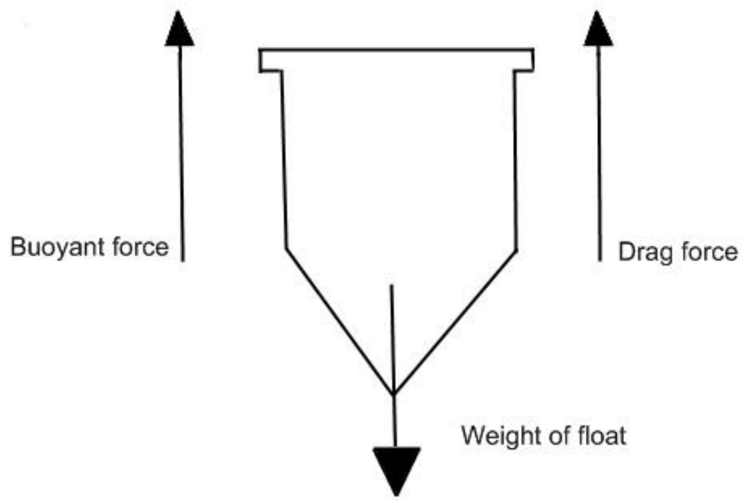


Figure F-2. Forces acting upon a rotameter float.

Mathematically, these forces are as follows:

$$Drag\ force = \frac{CA_f \varrho_m v^2}{2g_c}$$

$$Weight\ of\ float = \frac{V_f \varrho_f g}{g_c}$$

$$Buoyant\ force = \frac{V_f \varrho_m g}{g_c}$$

Where:

A_f = *cross sectional area of the float*
C = *drag coefficient*
g = *local acceleration due to gravity*
g_c = *dimensional constant*

v = *average gas velocity through the annular area of the meter*
V_f = *volume of the float*
ϱ_f = *density of the float*
ϱ_m = *density of the metered gas*

When the forces acting in an upward direction exactly equal the force acting in a downward direction, the float will remain stationary in the tube. Equating these forces yields:

$$\frac{CA_f\varrho_m v^2}{2g_c} + \frac{V_f\varrho_m g}{g_c} = \frac{V_f\varrho_f g}{g_c}$$

Cancelling like terms (g_c) and rearranging yields:

$$V_f\varrho_f g - V_f\varrho_m g = \frac{CA_f\varrho_m v^2}{2}$$

Solving for v and factoring out V_f and g from the first two terms yields:

(Eq. F-1)
$$v = \left[\frac{2V_f g\left(\varrho_f - \varrho_m\right)}{CA_f\varrho_m}\right]^{1/2}$$

The area of the float is equal to $\pi D_f^{\,2}/4$, where D_f is the diameter of the float. Substituting $\pi D_f^{\,2}/4$ for A_f in Equation F-1 yields:

(Eq. F-2)
$$v = \left[\frac{8V_f g\left(\varrho_f - \varrho_m\right)}{C\pi D_f^{\,2}\varrho_m}\right]^{1/2}$$

Let C_m equal $(8/C\pi)^{1/2}$, where C_m is called a meter coefficient and is dependent on the drag coefficient. Substituting C_m for $(8/C\pi)^{1/2}$ in Equation F-2 yields:

(Eq. F-3)
$$v = C_m\left[\frac{V_f g\left(\varrho_f - \varrho_m\right)}{D_f^{\,2}\varrho_m}\right]^{1/2}$$

Because the drag coefficient C is dependent on Reynolds Number, C_m must also be a function of Reynolds Number. Because the density of the gas flowing in the rotameter is very small compared to the density of the float, it can be ignored in the $(\varrho_f - \varrho_m)$ term. Modifying the $(\varrho_f - \varrho_m)$ term in Equation F-3 yields:

(Eq. F-4)
$$v = C_m \left[\frac{V_f g \varrho_f}{D_f^{\,2} \varrho_m} \right]^{1/2}$$

The volumetric flow rate (Q_m) through the rotameter is equal to the product of the velocity (v) and the annular area of the meter (A_m). Substituting Q_m/A_m for v in Equation F-4 yields:

$$\frac{Q_m}{A_m} = C_m \left[\frac{V_f g \varrho_f}{D_f^{\,2} \varrho_m} \right]^{1/2}$$

Rearranging terms and removing $D_f^{\,2}$ from the radical yields:

(Eq. F-5)
$$Q_m = \frac{C_m A_m}{D_f} \left[\frac{V_f g \varrho_f}{\varrho_m} \right]^{1/2}$$

The density of the float ϱ_f is equal to the mass of the float (m_f) divided by the volume of the float. Substituting m_f/V_f for ϱ_f in Equation F-5 and cancelling the V_f's yields:

(Eq. F-6)
$$Q_m = \frac{C_m A_m}{D_f} \left[\frac{g m_f}{\varrho_m} \right]^{1/2}$$

The density of the gas mixture passing through the meter (ϱ_m) is equal to $P_m M_m / R T_m$, where P_m is the absolute pressure at the meter, M_m is the apparent molecular weight of the gas mixture passing through the meter, R is the universal gas constant, and T_m is the absolute temperature of the gas mixture. Substituting $P_m M_m / R T_m$ for ϱ_m in Equation F-6 yields the general flow rate equation for a rotameter:

(Eq. F-7)
$$Q_m = \frac{C_m A_m}{D_f} \left[\frac{g m_f R T_m}{P_m M_m} \right]^{1/2}$$

Computation of Reynolds Number

Reynolds Number is defined as $v d \varrho / \mu$, where v is the velocity flow, d is a length which is characteristic of the physical system under study, ϱ is the density of the flowing fluid, and μ is the viscosity of the flowing fluid. When calculating Reynolds Number for a gas flowing through a rotameter, the length characteristic of the physical system (d) is the difference between the tube diameter (D_f) and the diameter of the float (D_r). Therefore, Reynolds Number may be calculated by using the following equation:

(Eq. F-8)
$$Re = \frac{v(D_r - D_f)\varrho}{\mu}$$

The average velocity of flow through the rotameter is given by Q_m / A_m where Q_m is the volumetric flow rate through the meter and A_m is the annular area between the inside circumference of the tube at the float position.

Substituting Q_m / A_m for v in Equation F-8 yields:

(Eq. F-9)
$$Re = \frac{Q_m(D_r - D_f)\varrho}{A_m \mu}$$

The density of the flowing fluid ϱ is equal to $P_m M_m / RT_m$, where P_m is the absolute pressure of the metered gas, M_m is the apparent molecular weight of the metered gas, R is the universal gas constant, and T_m is the absolute temperature of the metered gas.

Substituting $P_m M_m / RT_m$ for ϱ in Equation F-9 yields:

(Eq. F-10)
$$Re = \frac{Q_m(D_r - D_f)P_m M_m}{A_m \mu R T_m}$$

Adding the subscript *m* to the viscosity term μ in Equation F-10 to denote the viscosity of the metered gas yields the following equation, which is used to calculate Reynolds Number for gas flow in a rotameter.

(Eq. F-11)
$$Re = \frac{Q_m(D_r - D_f)P_m M_m}{A_m \mu_m R T_m}$$

F.4 Common Practices in the Use of a Rotameter for Gas Flow Measurement

It can be seen from Equation F-7 that the volumetric flow rate through a rotameter can be calculated when such physical characteristics as the diameter and the mass of the float and the annular area of the meter at each tube reading are known, providing measurements are made of the temperature, pressure, and molecular weight of the metered gas. Before these calculations of the volumetric flow rate can be made, data must be known about the meter coefficient, C_m. The meter coefficient being a function of Reynolds Number is ultimately a function of the conditions at which the meter is being used. To obtain data on the meter coefficient, the meter must be calibrated. However, because of the ease involved in using calibration curves, common practice is to use calibration curves to determine volumetric flow rates instead of calculating the flow rates from raw data.

Procedures for the Calibration of a Rotameter

A common arrangement of equipment for calibrating a rotameter is shown in Figure F-3.

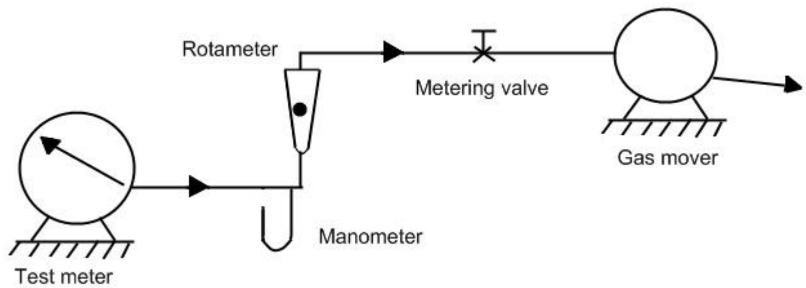


Figure F-3. Test setup for calibrating a rotameter.

Flow through the calibration train is controlled by the metering valve. At various settings of the rotameter float, measurements are made of the flow rate through the train and of the pressure and temperature of the gas stream at the rotameter. The temperature of the gas stream is usually assumed to be the same as the temperature of the ambient air. If the test meter significantly affects the pressure or temperature of the gas stream, measurements should also be made of the actual pressure and temperature at the test meter. A typical rotameter calibration curve is illustrated in Figure F-4.

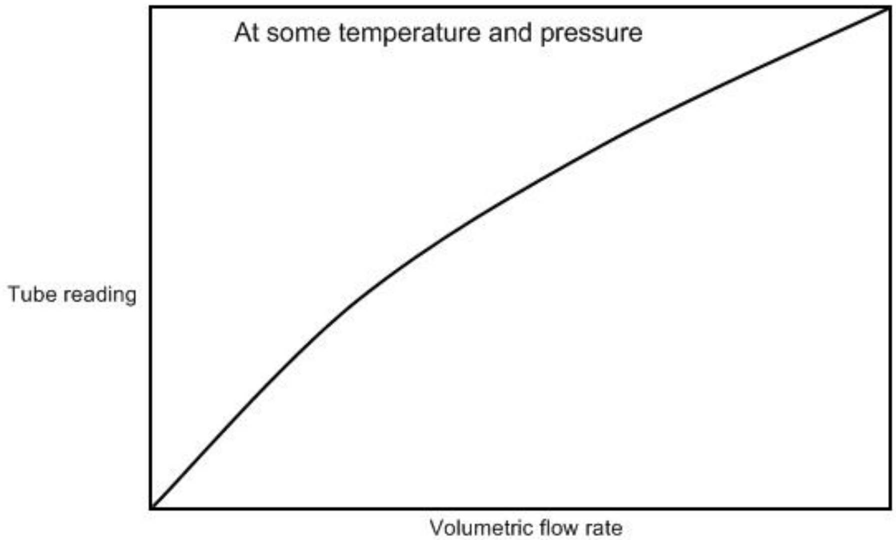


Figure F-4. Rotameter calibration curve.

To make the calibration curve useful, the temperature and pressure of the volumetric flow rate must be specified.

A Universal Calibration Curve

The normal arrangement of the components in a sampling train is shown in Figure F-5. Since the meter is usually installed downstream from the pollutant collector, it can be expected to operate under widely varying conditions of pressure, temperature, and molecular weight. This requires a different calibration curve for each condition of pressure, temperature, and molecular weight. This can be facilitated by drawing a family of calibration curves, which would bracket the anticipated range of pressures, temperatures and molecular weights, as shown in Figure F-6.

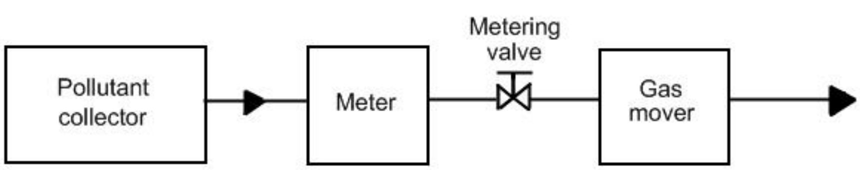


Figure F-5. Arrangement of sampling components.

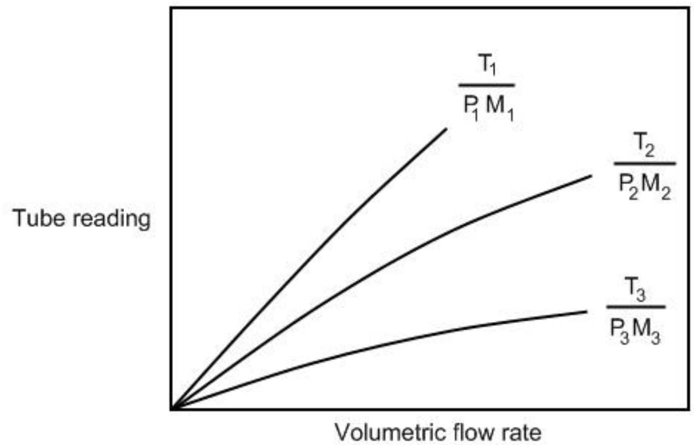


Figure F-6. Family of rotameter calibration curves.

Operation of a rotameter under extreme sampling conditions, particularly extreme temperatures, complicates the calibration setup. It is difficult, if not impossible, for most laboratories to be able to calibrate flow metering devices at high temperatures or unusual gas mixtures (especially where toxic gases are involved). For these reasons, it is desirable to develop a calibration curve which is independent of the actual expected sampling conditions. As previously mentioned, the flow through a rotameter is dependent upon the value of C_m, the meter coefficient (see Equation F-7), which is a function of the Reynolds Number for the flow in the rotameter. Therefore, to be independent of the sampling conditions, the calibration curve must be in terms of C_m and *Re*.

Development of a Universal Calibration Curve

Solving Equation F-7 for C_m gives the following relationship:

(Eq. F-12)
$$C_m = \frac{Q_m D_f}{A_m}\left(\frac{P_m M_m}{g m_f R T_m}\right)^{1/2}$$

Dividing Equation F-11 by Equation F-12 yields:

$$\frac{Re}{C_m} = \frac{\dfrac{Q_m\left(D_r - D_f\right)P_m M_m}{A_m \mu_m \mathrm{R} T_m}}{\dfrac{Q_m D_f}{A_m}\left(\dfrac{P_m M_m}{g m_f \mathrm{R} T_m}\right)^{1/2}}$$

Cancelling the like terms Q_m and A_m yields:

$$\frac{Re}{C_m} = \frac{\dfrac{(D_r - D_f)P_m M_m}{\mu_m RT_m}}{D_f\left(\dfrac{P_m M_m}{gm_f RT_m}\right)^{1/2}}$$

Simplifying:

$$\frac{Re}{C_m} = \left[\frac{(D_r - D_f)P_m M_m}{\mu_m RT_m}\right]\left[\frac{1}{D_f}\left(\frac{gm_f RT_m}{P_m M_m}\right)^{1/2}\right]$$

Combining the like terms P_m, M_m, and T_m yields:

(Eq. F-13)

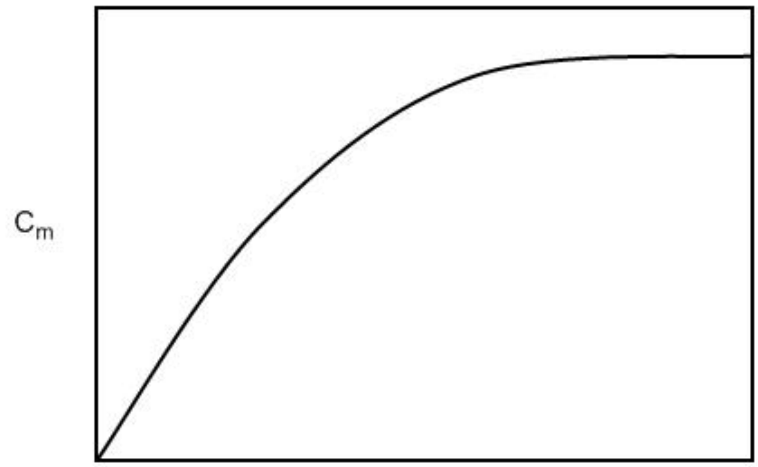
$$\frac{Re}{C_m} = \left[\frac{(D_r - D_f)}{\mu_m D_f}\right]\left(\frac{gm_f P_m M_m}{RT_m}\right)^{1/2}$$

Simplifying the $(D_r - D_f)$ and D_f relationship in Equation F-13 yields a dimensionless factor which has no limitations on either Reynolds Number or the meter coefficient C_m.

(Eq. F-14)

$$\frac{Re}{C_m} = \frac{1}{\mu_m}\left[\frac{D_r}{D_f}\right]\left(\frac{gm_f P_m M_m}{RT_m}\right)^{1/2}$$

A plot of the dimensionless factor Re/C_m defined by Equation F-14 versus the meter coefficient C_m as calculated from Equation F-12 on regular graph paper will yield a universal calibration curve which is independent of the sampling conditions. Such a plot is illustrated in Figure F-7.



$$\frac{Re}{C_m} = \frac{1}{\mu_m}\left[\left(\frac{D_r}{D_f}\right) - 1\right]\left(\frac{g m_f P_m M_m}{R T_m}\right)^{1/2}$$

Figure F-7. A universal calibration curve for a rotameter.

F.5 Use of the Universal Calibration Curve for a Rotameter

To Determine an Existing Flow Rate

To determine an existing flow rate, measurements must be made of the gas temperature and pressure as well as the float position. Data from the manufacturer of the rotameter will yield information on the diameter of the tube at the various float positions and on the diameter and mast of the float. The apparent molecular weight of the gas being metered can be calculated if the composition of the gas stream is known. The viscosity of the gas stream can be determined if the temperature of the gas stream is known (see Perry's *Chemical Engineer's Handbook*). From this data the Re/C_m factor (see Equation F-14) can be calculated. The universal calibration curve is then entered at the calculated value of Re/C_m and the corresponding C_m is noted. Q_m is then calculated from Equation F-7.

To Establish a Required Sampling Rate

To establish a required sampling rate, estimates are made of the metered gas pressure (P_m), the metered gas temperature (T_m), the apparent molecular weight of the metered gas (M_m), and the area of the meter (A_m) which will exist at the desired sampling rate. Using these estimated values, the meter coefficient, C_m, is calculated (see Equation F-12) for the desired sampling rate Q_m. The universal calibration curve (see Figure F-7) is entered at this value of C_m and the corresponding factor is noted. $\left[\left(D_r/D_f\right)-1\right]$ is solved by using the following equation which is a rearrangement of Equation F-14:

(Eq. F-15)
$$\left[\left(\frac{D_r}{D_f}\right)-1\right]=\mu_m\left(\frac{Re}{C_m}\right)\left(\frac{RT_m}{gm_f P_m M_m}\right)^{1/2}$$

The float position can be determined from the value $\left[\left(D_r/D_f\right)-1\right]$. For some rotameters the value of $\left[\left(D_r/D_f\right)-1\right]$ is the tube reading divided by 100. If the area of the meter corresponding to this float position is not equal to the original estimated value for the meter area, the new value of area is used as an estimate and the entire procedure is repeated until the estimated area and the calculated area are equal. Then upon setting the float position at this tube reading, T_m, P_m, and M_m, are noted. If they are different from the original estimates, the procedure is repeated using the observed values of T_m, P_m, and M_m as estimates. Experience will aid in selecting original estimates that are nearly accurate so that the required sampling rate may be set fairly rapidly.

To Predict Calibration Curves

The above techniques are very cumbersome to apply in the field and, as a result, the universal calibration curve should not be used in such a manner.

The real utility of the universal calibration curve is that it can be used to predict calibration curves at any set of conditions. This results in a great reduction in laboratory work in that the rotameter need only be calibrated once and not every time the conditions at which the meter is operated change.

The first step in predicting calibration curves from the universal calibration curve of a rotameter is to ascertain the anticipated meter operating range for the sampling application of concern. Once this operating range is established, an arbitrary selection of a point on the universal calibration curve is made (see point in Figure F-8). The coordinates of point a, point b $\left(Re/C_m\right)$, and point c (C_m) are determined. Values of T_1, P_1, and M_1 and the value of Re/C_m are used to calculate a value for $\left(D_r/D_f\right)-1$ by means of the following equation:

(Eq. F-15)
$$\left[\left(\frac{D_r}{D_f}\right)-1\right]=\mu_m\left(\frac{Re}{C_m}\right)\left(\frac{RT_m}{gm_f P_m M_m}\right)^{1/2}$$

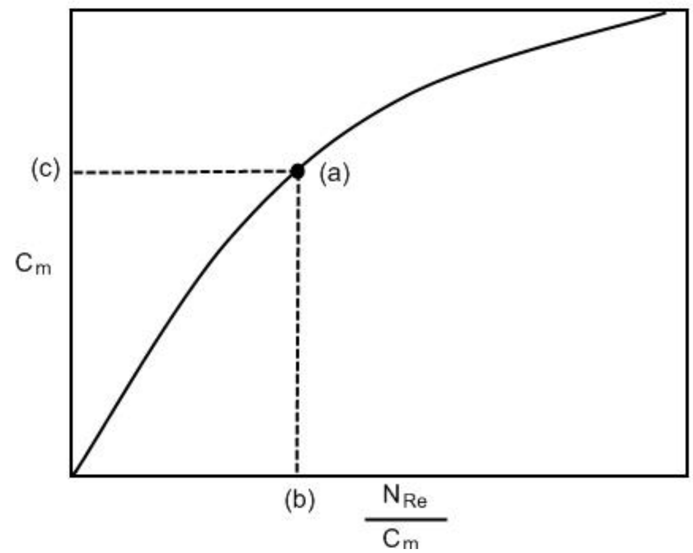


Figure F-8. Predicting calibration curves from the universal calibration curve (N_{Re}=Re, or Reynolds Number).

The area of the meter (A_m) is calculated from this value of $\lfloor (D_r/D_f) - 1 \rfloor$ and is used along with the assumed values of T_1, P_1, and M_1 and the value of C_m from the universal calibration curve to calculate a volumetric flow rate by means of the following equation:

(Eq. F-7)

$$Q_m = \left(\frac{C_m A_m}{D_f} \right) \left[\frac{gm_f RT_m}{P_m M_m} \right]^{1/2}$$

This procedure is repeated until enough points are available to plot a normal calibration curve. The entire procedure is repeated using new values for temperature, pressure, and molecular weight until a family of calibration curves is plotted. Of course, this family of curves should bracket the anticipated meter operating conditions for the sampling application of concern. The volumetric flow rate (Q_m) is plotted versus either the area of the meter (A_m) or the tube reading that corresponds to the meter area.

Field operation is greatly simplified if the tube reading is used. A typical family of calibration curves is shown in Figure F-9.

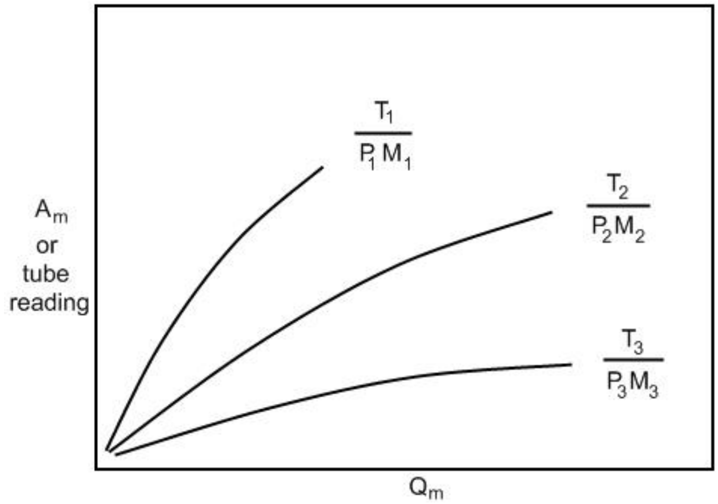


Figure F-9. Calibration curves predicted from universal calibration curve.

Notice that these curves are similar to the calibration curves illustrated in Figure F-6. The difference between them is the manner in which they were obtained. The curves of Figure F-6 were obtained by an actual laboratory calibration run for each set of conditions illustrated, whereas the curves of Figure F-9 were obtained by mathematical manipulation of data from only one calibration run. This can, of course, save considerable laboratory time. In addition, it may not be possible to ascertain, in the laboratory, calibration data at extreme conditions, particularly at high temperatures.